B.Sc V Semester (NEP) Unit III Quantum Mechanics

Syllabus: Introduction to Quantum Mechanics: Brief explanation on failure of classical physics: black body radiation, Photoelectric effect, Compton Effect, stability of atoms and spectra of atoms, variation of specific heat of solids with temperature. Planck's quantum theory: a brief discussion. Compton scattering: Expression for Compton shift (With derivation). Mention of expression for scattered photon energy and K.E. of recoil electron.

Matter Waves: de Broglie hypothesis of matter waves, Wave description of particles by wave packets, Group and Phase velocities and relation between them, Experimental evidence for existence of matter waves: G.PThomson's experiment and its significance.

Heisenberg Uncertainty Principle: Statement and elementary proof of Heisenberg's relation between momentum and position and mention of uncertainty principle for energy & time, and angular momentum & angular position, illustration of uncertainty principle by Diffraction of electrons through single slit experiment. Explain why electron cannot exist inside the nucleus based on uncertainty principle.

Introduction:

Newtonian mechanics, Maxwell's electromagnetic theory and thermodynamics guided the growth of science and engineering during the years spanning 17th to 19th centuries. The theories explained almost all the scientific results of those times and it seemed nothing more could be added. The above theories which are successful in the realms of macroscopic world are regarded as classical physics.

In the classical physics, matter and fields are treated as entirely independent entities. Macroscopic particles move and interact according to Newton's laws.

According to classical wave theory,

- Electromagnetic waves are generated by accelerated charges.
- The waves spread out continuously through the space and the wave energy is not localized; but is distributed over the volume of the wave.
- The energy of the wave is not related to frequency and is proportional to the square of the amplitude of the wave.
- The thermodynamic equilibrium of an assembly of neutral particles is governed by Maxwell Boltzmann statistics.

QUANTUM MECHANICS:

Quantum (wave) mechanics is the department of theoretical physics dealing with the laws of motion of particles in the microcosm region (10^{-8} to 10^{-15}m). For the motion of particles at velocities v<< c, where c is the velocity of light in a vacuum, non-relativistic quantum mechanics is applied; at v~c, it is replaced relativistic quantum mechanics. The objects studied by wave mechanics are crystals, molecules, atoms, atomic nuclei and elementary particles.

The wave theory of light successfully explained the phenomenon resulting due to distribution of light energy in space when it passes across apertures, obstacles etc. Such as interference, diffraction; but it could not explain the phenomenon resulting due interaction of light with matter such as blackbody spectrum, photoelectric effect, Compton effect, emission and absorption of radiation. The failure of classical physics i.e., Newtonian laws, thermodynamic and wave theory of light to explain the phenomenon resulting due to interaction, of light with matter gave birth to quantum theory of light.

Quantum theory was first introduced by Max Planck of Germany in the year 1900 while trying to explain the observed energy distribution of the electromagnetic radiation emitted by a blackbody. Before understanding the laws of quantum mechanics, it is better to consider some experimental situations and theoretical explanations, failures and inspired guesses.

1. Stability of the Atom:

In 1804, Dalton proposed his atomic theory. According to this theory all matter was composed of very small particles called **atoms**. He believed that the atoms of the same substance were all alike and differed from those of all other substances and further, that the atoms could not be subdivided, destroyed or created. **Prout** proposed that all the substances were made of hydrogen atoms**. J. J. Thomson** in 1898, proposed an atom model where the atom was supposed to be a sphere filled with positively charged matter of uniform density in which just sufficient number of electrons were embedded to balance the positive charge. This atom model is referred to as **plum-pudding model.** Thomson atom model explains the phenomena like thermionic emission, photoelectric emission and also generation of electromagnetic waves. But it failed to account for the results of α -ray scattering experiment and also like line spectra given out by the atoms.

It was **Rutherford**, who based on his experimental results on scattering of α-particles by matter, proposed nuclear model of the atom. According to nuclear model of the atom, the atom has a massive positively charged central core in a small volume called the **nucleus**. The electrons revolve around the nucleus in almost circular orbits. The dimension of the atom is of the under of 10^{-10} m and that of nucleus is about 10^{-15}m. Thus, the atom, with its electrons orbiting round the nucleus, resembles our solar system. The centripetal force which is necessary for the circular motion of electrons is provided by the electrostatic force of attraction between nucleus and electrons. But, according to Maxwell's electromagnetic theory, an orbiting electron must radiate energy. Such electrons will continuously loss energy, finally falling into the nucleus. Rutherford's model could not explain the stability of the electron orbit. Also, this model could not explain the observed line spectrum of hydrogen atom. Thus, in a classical planetary model, an electron orbit would be unstable, with the electron spiraling into the nucleus as it radiates electromagnetic energy. The leads to the conclusion that **classical theory fails to explain the stability of the atom.**

2. Atomic Spectra:

According to Bohr, an atom does not radiate when in a stationary state. Radiation is emitted, in the form of a photon, only when an atom undergoes a transition from one stationary state to another one of lower energy. If a photon of energy hν is emitted in the transition, then by conservation of energy,

$$E_2 = E_1 + h\nu \qquad \text{or} \qquad h\nu = E_2 - E_1 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If the energies are expressed in terms of n_1 and n_2, the frequency of emitted spectral line is given by

$$h\nu = -\left[\frac{mZ^2e^4}{8\epsilon_0^2h^2n_2^2} - \frac{mZ^2e^4}{8\epsilon_0^2h^2n_1^2}\right] \quad \text{Or} \quad h\nu = \left[\frac{mZ^2e^4}{8\epsilon_0^2h^2n_1^2} - \frac{mZ^2e^4}{8\epsilon_0^2h^2n_2^2}\right] \quad \text{Or}$$

$$\nu = \frac{mZ^2e^4}{8\epsilon_0^2h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \text{Since} \quad \nu = \frac{c}{\lambda} \quad \text{then in terms of wave number}$$

$$\bar{\nu} = \frac{1}{\lambda} = Z^2\,R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where, $R = \frac{me^4}{8\epsilon_0^2ch^3}$, the **Rydberg Constant**

n_1 and n_2, are two different states such that $n_2 > n_1$. The hydrogen atom emits a spectrum consisting of different series and each series containing discrete spectral lines.

Zeeman observed a magnetic-optical phenomenon, in which spectral lines are affected by an applied magnetic field and split into several components. Zeeman analyzed the splitting of the sodium D-lines and found that $\frac{e}{m}$ of the particles responsible. The quantitative description of Bohr's concepts and Zeeman splitting constitute spectacular successes of modern quantum mechanics.

But according to the classical mechanics, an excited atom should emit electromagnetic radiation continuously of all possible wavelengths. Since this is in contradiction with the experimental results, **classical mechanics does not hold good to explain the formation of spectral line in the case of atoms.**

3. Blackbody Radiation:

The quantum idea first originated from an attempt to explain the spectral distribution of energy radiated by a blackbody. It is well-known that when a body is heated it emits electromagnetic radiation. **A blackbody is defined as one which absorbs all the radiation falling upon it**. As the radiating power of a body is proportional to its absorbing power, a blackbody would also radiate more strongly, at any given temperature, than any other surface. The radiations are independent of the nature of the body and depend only on the temperature of the blackbody. The energy distribution in the radiation spectrum of a blackbody is of the form of the curves in below figure. The different curves correspond to different temperatures of the blackbody. The following conclusions can be obtained:

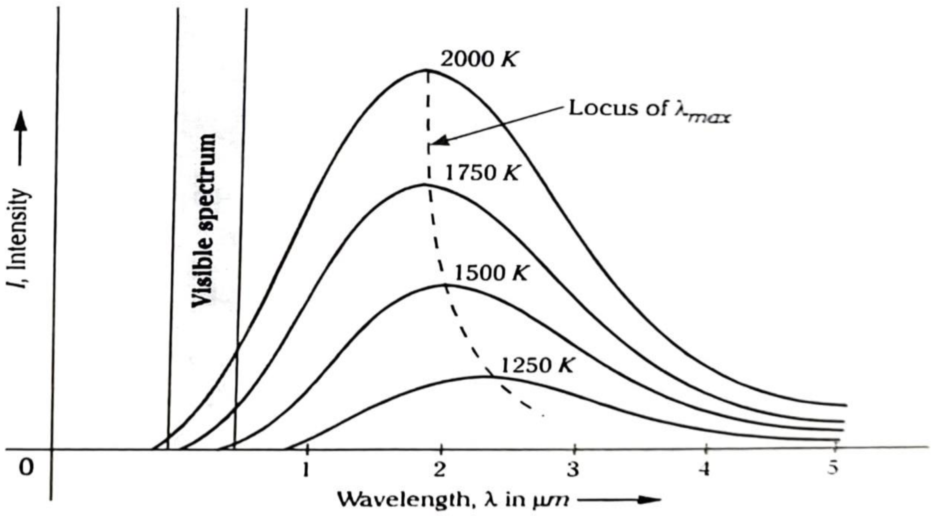
i) At a given temperature, the spectral radiancy has a single peak.

ii) As the temperature increases, the area under each curve increases. It shows that the rate of emission increases very rapidly as the temperature rises.

iii) As the temperature increases, the peak wavelength λ_m for which the energy is maximum is found to shift progressively towards the shorter wavelengths.

iv) At a given temperature the energy is not uniformly distributed in the spectrum.

Attempts were made to explain the fundamental laws regarding the nature of the curves were established



Energy distribution in spectrum of a blackbody at various temperatures

a) Stefan proposed fourth power law, which states that the rate at which a perfectly black body of unit area emits radiations is directly proportional to the fourth power of its absolute temperature.

If E is the heat energy radiated per unit area of the blackbody and T is its temperature, according to Stefan's law, $E \propto T^4 \quad Or \quad E = \sigma T^4$ ………………..(1).

Where σ is called Stefan's constant and its value is 5.672 x 10^{-8} $Wm^{-2}K^{-4}$ This has been theoretically verified by Boltzmann using thermodynamic principles. This law explains only increased energy with increase in temperature but not the distribution of energy in the blackbody radiation spectrum.

b) According to Wien's displacement law, wavelength (λ_m) of radiation corresponding to the maximum intensity is inversely proportional to the temperature, T. Then

$\boldsymbol{\lambda_m \propto \frac{1}{T}}$ Or $\boldsymbol{\lambda_m T = a\ constant = b}$

Value of the constant b, called Wien's constant has been found to be 2.892 x 10^{-3} mK. The dotted line in Fig. 1 represents $\boldsymbol{\lambda_m}$ at various temperatures.

Wein also showed that the maximum energy E_m in directly proportional to the fifth power of the temperature i.e,

$$\boldsymbol{E_m \propto T^5} \qquad \text{or} \qquad \boldsymbol{E_m = constant\ T^5}$$

The constant value was determined by Wien and gave the following relation:

$$\boldsymbol{E_\lambda = a\lambda^{-5}\, e^{-b/\lambda T}}$$

Where a and b are constants. This equation represents the Wein's law of distribution of energy.

c) Rayleigh and Jeans deduced a radiation formula based on classical physics. It is given by the relation,

$$E_{\lambda} = \frac{8\pi kT}{\lambda^4}$$

The Rayleigh-Jeans formula agrees with experiments only at low frequencies; at higher frequencies it is clearly wrong because it increases without bound (as v^2). Classical physics was thus unable to account for this important experimental phenomenon of the emission by a blackbody.

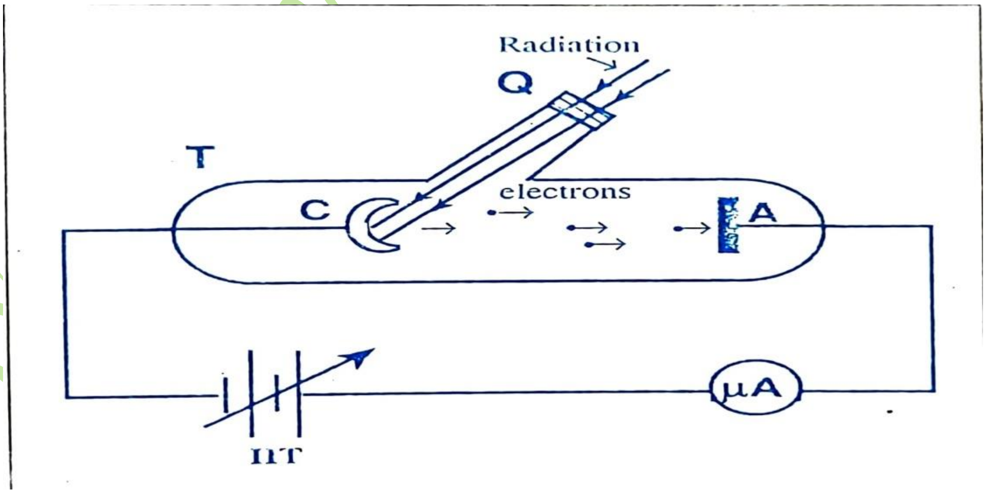
In 1900 Max Planck presented a formula that could describe the measured frequency distribution of blackbody radiation.

PHOTOELECTRIC EFFECT:

Photoelectric effect is the phenomenon of emission of electrons from the surface of metals when radiation of suitable frequency is incidental on them. The electrons **emitted** in this phenomenon are called photoelectrons and the current produced due to these electrons is called **photoelectric current.** The phenomenon of photoelectric emission was discovered by **Heinrich Hertz** during electromagnetic wave experiments.

Experimental study of photoelectric effect:

The apparatus consists of an evacuated glass/quatz tube T which encloses a photosensitive plate C (emitter) and a metal plate A (collector). Monochromatic light from a source S passes through the window W and falls on the photosensitive plate C. Electrons are emitted by the plate C and they are collected by the plate A because of the electric field created by the battery B. The battery maintains a potential difference between the plate C and A which can be varied.



The polarity of the plates C and A can be varied. The polarity of the plates C and A can be reversed by a commutator K. Thus, plate A can be maintained at a desired positive potential with respect to the emitter C, the electrons emitted by C are attracted towards A. This causes an electric current in the circuit which is measured by the micro ammeter μA. The potential difference between emitter and collector plates is measured by a voltmeter V. The frequency and the intensity of incident light can be varied.

Law of photoelectron emission:

- The photoelectric emission is an instantaneous process. (Time lag between incidence of photon & electron emission is nearly 10^{-9} s or less, even when the incident radiation is made very dim).
- For a given metal, there exists a certain minimum frequency of the incident radiation below which no emission of photoelectrons taken place. This minimum frequency is called **threshold frequency.**
- For a given metal and frequency of the incident radiation (above the threshold frequency), the number of photoelectrons ejected per second (photoelectric current) is directly proportional to the intensity of incident radiation.
- Above the threshold frequency, the maximum kinetic energy (or the stopping potential) of the emitted Photo-electrons increases linearly with the frequency of the incident radiation, but it is independent of its.
- The kinetic energy of the emitted electrons have values between zero and a definite maximum.

Einstein's photoelectric equation:

Based on Max Planck's quantum hypothesis, Albert Einstein in 1905 gave a simple and complete explain of photoelectric effect on the basis of quantum theory of radiation. He attributed the particle nature of photon. According to him when photon interacts with matter it behaves as a particle and photoelectric effect is due to the collision between incident photon and free electron in the metal. During the collision the electron absorbs photon and gains an energy E= hv(Photon energy) .

We know that free electrons are bound to the material. They are free to move randomly inside the material. They are stay within the metal due to surface force. Hence certain amount of energy is necessary to free the electrons from the surface of the material. The minimum energy needed for the electron to escape from the metal surface is called **work function (ϕ_o).** It is constant for a given material and is measured in **electron volt (eV).**

When light is incident on a metal, the photons having energy hv collide with electrons at the surface of the metal. During these collisions, the energy of the photon is completely transferred to the electron. If this energy is sufficient, the electrons are ejected out of the metal instantaneously. If the energy hv of the incident photon exceeds the work function ϕ_o, the electrons are emitted with a maximum kinetic energy.

$$K_{max} = hv - \phi_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If v_{max} is the maximum velocity of photoelectrons emitted, we have

$$\frac{1}{2} m v_{max}^2 = hv - \Phi_0 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This equation is known as **Einstein's Photo electronic Equation.**

The experimental features of the photoelectric effect discussed above were not clearly established when Einstein purposed the photon interpretation of light in 1905. The linear dependence of the stopping potential on frequency was confirmed by careful experiments in 1916 by R.A. Millikan. The above experimental facts cannot be explained on the basis of classical electromagnetic theory. Calculations showed that when ultraviolet of wavelength $4000A^0$ incident on sodium it takes nearly 500 days to dislodge a photoelectron. But, photoelectric effect is an instantaneous phenomenon. Hence, the classical electromagnetic theory failed to explain photoelectric emission.

5. Compton Effect:

In 1923 A.H. Compton found that the frequency of some of the X-rays scattered by electrons was not the same as the frequency of the incident X-rays.

- The scattered X-rays consist of two components. One component has the same wavelength as the original incident X-rays and the other component has a slightly longer wavelength. **The phenomenon in which there is change in wavelength of the scattered X-rays is called Compton Effect.** Compton showed the interaction can be interpreted as collision of two particles, a photon and an electron. The change in wavelength in the Compton effect depends on the scattering angle. In the process, both momentum and energy are conserved.
- The distribution of the scattered intensity is not symmetrical.
- The scattering constant was found to depend on the wavelength of X-rays.

According to classical electromagnetic theory,

- When X-rays incident on atoms of an element the electrons are set into forced vibrations of same frequency as that of incident radiation. These vibrating electrons emit the radiations of same frequency. Hence there is no change in wavelength of the scattered radiations.
- The distribution of intensity of X-rays should be symmetrical.
- The scattering constant should have a constant value of 0.2 and it should be independent of the wavelength of the incident X-rays.

Hence, classical electromagnetic theory failed to explain the experimentally observed facts. This phenomenon was successfully explained by A.H. Compton on the basis of quantum theory proposed by Max Planck.

6. Specific Heat of Solids:

The specific heat of a substance is defined as the amount of heat required to increase the temperature of 1 kg of the substance through one degree Kelvin.

The specific heat depends on the nature of the substance, but it is independent of the mass of the system. The heat required to change the temperature of a system depends on the process. The specific heat increases with raise in temperature and it tends to zero when the temperature reaches absolute zero. It also shows temperature dependence at lower temperatures. The variation of specific heat of solid with temperature has been explained by Dulong and Petit's law. It states that, at room temperature the product of specific heat and atomic weight of a solid is a constant for all solids and is approximately equal to 6.4.

$$Atomic\ Weight\ X\ Specific\ heat = 6.4$$

The product of specific heat and atomic weight of a solid is also called the **atomic heat.** Since atomic weight is constant for the given solid, its specific heat should also be constant at all temperatures. But, the experiment performed by Debye shows that the specific heat of a solid is constant at higher temperatures but it decreases with decreasing temperature and becomes zero at absolute zero. Hence classical theory fails completely at very low temperatures. Albert Einstein was the first to explain the decrease of the specific heat of solids of low temperature on the basis of quantum theory.

Compton Effect:

Compton discovered that when X-rays of sharply defined frequency were incident on a material of low atomic number such as carbon or graphite, they suffer a change of frequency on scattering. The scattered beam consists of two wavelengths; in addition to the incident wavelength 'λ' there must be a line of longer wavelength .The change in wavelength is due to loss in energy of the incident X-ray. This elastic interaction is known as **Compton scattering**.

Theory of Compton Effect: To explain the effect, Compton applied Einstein's quantum theory of light with the assumption that the incident photon possesses momentum.

Following are the postulates on which the theory was based.

1) A beam of monochromatic X-ray of frequency 'ν' consists of stream of photos each of energy $E = h\gamma$ and momentum $p = \frac{h\nu}{c}$
2) Scattering of X-rays by atoms of an element is the result of elastic collision between photon and atomic electron.

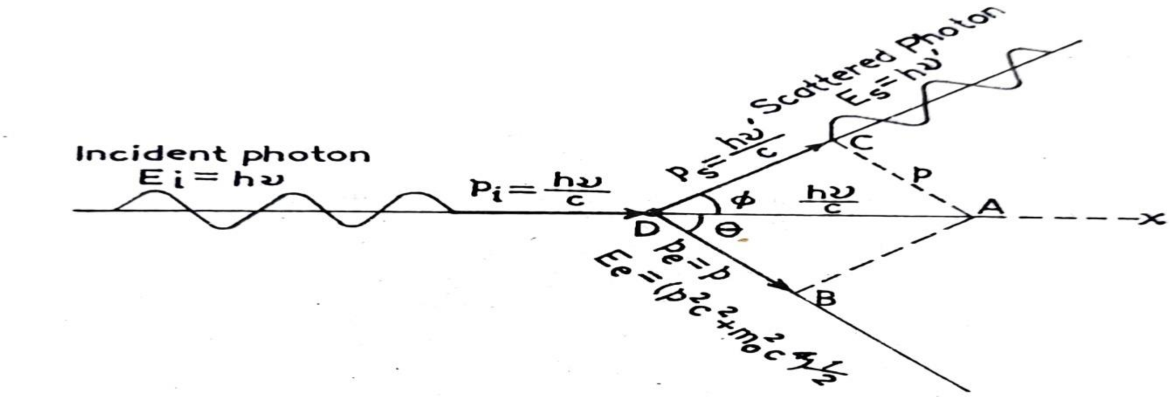


Figure (1) :COMPTON SCATTERING

Let a beam of monochromatic X-ray of frequency 'ν 'and wavelength 'λ' is incident on a target O which is an element of low atomic number such as carbon or graphite block. Suppose the incident photon makes a perfectly elastic collision with an electron initially at rest. Suppose the photon is scattered through an angle φ and the electron moves in the direction θ as shown in figure (1).

Let ν^1 be the frequency of the scattered photon, p be the momentum of the recoil electron and m_0 be the rest mass then we have, The energy of the incident photon = hν and its momentum = $\frac{h\nu}{c}$

The energy of the scattered photon = $h\nu^1$ and its momentum = $\frac{h\nu^1}{c}$

The energy of the electron at rest (rest mass energy) = m_0c^2 and its momentum = 0

The relativistic energy of the electron having momentum p after collision = $\sqrt{p^2c^2 + m_0^2c^4}$

From the principle of conservation of energy, we have

Total final energy = Total initial energy

$h\nu^1 + \sqrt{p^2c^2 + m_0^2c^4} = h\nu + m_0c^2$

$\sqrt{p^2c^2 + m_0^2c^4} = h\nu - h\nu^1 + m_0c^2$

$\sqrt{p^2c^2 + m_0^2c^4} = h(\nu - \nu^1) + m_0c^2$

Squaring on both sides

$p^2c^2 + m_0^2c^4 = h^2(\nu - \nu^1)^2 + 2h(\nu - \nu^1)m_0c^2 + m_0^2c^4$

$\frac{p^2c^2}{h^2} = (\nu - \nu^1)^2 + \frac{2m_0c^2}{h}(\nu - \nu^1)$ -- (1)

Momentum is a vector quantity and is conserved in collision between the two bodies on each of the two mutually opposite direction. In this case resolve the momentum along and at right angles to the direction of incident photon, we get.

(i) In the direction of incident photon

$\frac{h\nu^1}{c}\cos\varphi + p\cos\theta = \frac{h\nu}{c}$ Multiply both sides by $\frac{c}{h}$ we get

$\nu^1\cos\varphi + \frac{pc\cos\theta}{h} = \nu$

$$\frac{pc\cos\theta}{h} = \nu - \nu^1\cos\varphi \ldots\ldots\ldots\ldots\ldots\ldots. (2)$$

(ii) In the direction at right angles to the direction of incident beam

$p\sin\theta - \frac{h\nu^1}{c}\sin\varphi = 0$ Multiply both sides by $\frac{c}{h}$ we get

$\frac{pc\sin\theta}{h} = \nu^1\sin\varphi$------------------------- (3)

To eliminate 'θ' squaring equations (2) and (3) and adding

$\frac{p^2c^2}{h^2} = (\nu - \nu^1\cos\varphi)^2 + \nu_1^2\sin^2\varphi$

$= \nu^2 - 2\nu\nu^1\cos\varphi + \nu_1^2\cos^2\varphi + \nu_1^2\sin^2\varphi$

$= \nu^2 - \nu\nu^1\cos\varphi + \nu_1^2$

$\frac{p^2c^2}{h^2} = (\nu - \nu^1)^2 + 2\nu\nu^1 - 2\nu\nu^1\cos\varphi$

$\frac{p^2c^2}{h^2} = = (\nu - \nu^1)^2 + 2\nu\nu^1(1 - \cos\varphi)$ -----------------(4)

Equating equations (1) and (4) we get

$\frac{m_0c^2}{h}$ $(\nu - \nu^1) = \nu\,\nu^1\ (1 - \cos\varphi)$

$\frac{\nu-\nu^1}{\nu\nu^1} = \frac{h}{m_0c^2}(1 - \cos\varphi)$ Or $\frac{1}{\nu^1} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1 - \cos\varphi)$

Express in terms of wavelength using, $\nu = \frac{c}{\lambda}$ and $\nu^1 = \frac{c}{\lambda^1}$

$\frac{\lambda^1}{c} - \frac{\lambda}{c} = \frac{h}{m_0c^2}(1 - \cos\varphi)$

$\lambda^1 - \lambda = \frac{h}{m_0c}(1 - \cos\varphi)$ Or $\boldsymbol{\Delta\lambda = \frac{h}{m_0c}(1 - \cos\varphi)}$ ----(6)

$\boldsymbol{\Delta\lambda = \frac{2h}{m_0c}\sin^2\varphi/2}$ **----(7)**

Equation (6) and (7) are the expression for Compton shift in terms of wavelength of the X-rays scattered by the electron in the low atomic number element like graphite**.**

The numerical value of the quantity $\lambda_c = \frac{h}{m_0c} = \frac{6.625\times10^{-34}}{9.11\times10^{-31}\times3\times10^8}$

= **0.02426 A^0**is known as **Compton wavelength**.

The energy of a photon which has a Compton wavelength is

$E = h\nu = \frac{hc}{\lambda_c} = \frac{hc}{h/m_0c} = m_0c^2$

Thus, the Compton wavelength of an electron is the wavelength of radiation, whose photon has energy equal to rest energy m_0c^2 of the scattered electron.

Conclusion: From equation (6) we have

i) If $\varphi = 0$,then $\lambda^1 - \lambda = 0$.There is no scattering along the direction of incident photon,there is no Compton shift.

ii) If $\varphi = \pi/2$, then $\lambda^1 - \lambda = \Delta\lambda = \frac{h}{m_0c}$ **= 0.02426 A^0**

iii) If $\varphi = \pi$, then $\lambda^1 - \lambda = \Delta\lambda = \frac{2h}{m_0c}$ **= 0.04852 A^0**

Thus, as φ varies from 0 to π ,the wavelength of the scattered photon varies from λ to $\lambda + \frac{2h}{m_0c}$

iv) The change in wavelength is independent of incident wavelength**.**

v) The change in wavelength has same value for all the substance containing free electrons.

vi) The change in wavelength depends only on the scattering angle.

vii) The change in wavelength is maximum when $\varphi = \pi$

Kinetic energy of recoil electron

The KE of the incident photon= $h\nu$, and KE of the scattered photon = $h\nu^1$

Therefore kinetic energy of recoil electron is the decrease in energy of the scattered photon

Ie $E = h\nu - h\nu^1$

$E = h(\nu - \nu^1) = h\nu\left(1 - \frac{\nu^1}{\nu}\right)$

$= h\nu\,(1 - \frac{\lambda}{\lambda^1}) \quad = h\nu\,(\frac{\lambda^1-\lambda}{\lambda^1}) \quad = h\nu\left[\frac{d\lambda}{\lambda+d\lambda}\right] \quad (Since\ \nu = \frac{c}{\lambda})$

$$E = \frac{h\nu \times \frac{h}{m_0\,c}(1-cos\emptyset)}{\lambda + \frac{h}{m_0\,c}(1-cos\emptyset)} = \frac{h\nu \times \frac{h}{m_0\,c\lambda}(1-cos\emptyset)}{1 + \frac{h}{m_0\,c\lambda}(1-cos\emptyset)} = \frac{h\nu\alpha\,(1-cos\emptyset)}{1+\ \alpha\,(1-cos\emptyset)} \qquad \text{since } \frac{h}{m_0\,c\lambda} = \alpha$$

The energy of the recoil electron

$E = 0$ when $\emptyset = 0$ and $E = \frac{2h\nu\,\alpha}{1+2\,\alpha}$ when $\emptyset = \pi$

∴ The maximum energy that athe X-ray photon can transfer to the electron is $\frac{2h\nu\alpha}{1+2\,\alpha}$ which is less than hγ as α is a positive quantity .Hence the photon can transfer all of its energy to the electron.

MATTER WAVES

Wave and Particle Duality of Matter: In 1924, Louis de Broglie of France put forward the suggestion that, matter, like radiation has dual nature i.e., matter which is made up of discrete particles, atoms, protons, electrons etc., might exhibit wavelike properties under appropriate conditions. The existence of de Broglie waves was experimentally demonstrated in 1927, and the duality principle they represent provided the starting point for Schrodinger's successful development of quantum mechanics. The concept of wave nature of matter arose from the dual character of radiation which sometimes behaves as a wave and at other times as a particle.

1. Radiations including visible light, infra-red, ultraviolet and X-rays etc., behave as waves in propagation experiments based on interference and diffraction. These experiments conclusively prove the wave nature of these radiations because they require the presence of two waves at the same position at the same time. Obviously, it is impossible for the two particles to occupy the same position at the same time.

2. Radiation behaves as a particle in interaction experiments which include black-body radiation, photoelectric effect and Compton Effect. Here, radiation interacts with matter in the form of photons or quanta. ***Note that radiation cannot exhibit its particle and wave properties simultaneously.***

Still, this dual nature of radiation was not easily accepted because of the apparently contradiction

A wave is specified by its (a) frequency (b) wavelength (c) phase or wave velocity d) amplitude and (e) intensity. Moreover, a wave spreads out and occupies a relatively large region of space.

A particle is specified by (a) mass (b) velocity (c) momentum and (d) energy. Moreover, a particle occupies a definite position in space and hence is very small.

In view of the above, it is rather difficult to accept the conflicting ideas that radiation is a wave which is spread out over space and also a particle which is localized at a point in space. However, this acceptance is essential if one has to satisfactorily explain the result of experiments which can be performed with radiation.

Particle nature of light:

Photoelectric effect proved that light in interaction with matter behaved as it was made of quanta or packets of energy, each of energy hν. Einstein showed that the light quantum can also be associated with momentum $\frac{h\nu}{c}$. A definite value of energy as well as momentum is a strong sign that the light quantum can be associated with a particle. This particle later named as **photon.**

Summary of Photon Picture of Electromagnetic Radiation:

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called photons.

(ii) Each photon has energy $E = h\nu = \frac{hc}{\lambda}$ and momentum $p = \frac{h\nu}{c} = \frac{h}{\lambda}$ where h is Planck's constant, ν is the frequency and λ is the wavelength of the radiation and c is the velocity of light.

(iii) The photon energy and momentum depend only on frequency of the radiation it is independent of intensity of the radiation.

(iv) Photons are electrically neutral and are not deflected by electric or magnetic fields.

(v) All photons emitted from a source travel with same velocity through space.

(vi) When photon collides with particle, the total energy and total momentum are conserved. However, the number of photons may not be conserved in a collision. The photon may be absorbed or a new photon may be created.

(vii) The rest mass of a photon is zero.

Wave Nature of Matter:

Light exhibits the phenomenon like reflection, refraction, interference, diffraction and polarization. These phenomena can be explained satisfactorily by treating light as waves (electromagnetic waves). Light also exhibits phenomena such as photoelectric effect and Compton effect. These phenomena can be explained by treating light as particles in the form of quanta or photons. Thus, light exhibits dual nature, sometimes behaving as waves and sometimes behaving like particles. (But light does not exhibit wave nature and particle nature simultaneously)

If radiation has a dual (wave-particle) nature, particles of nature (the electrons, protons, neutrons etc.) also exhibit wave-like character. Louis Victor de Broglie gave a hypothesis that moving particles of matter should display wave-like properties under suitable conditions. He reasoned that Nature manifests as matter and radiation. If radiant energy shows dual nature, (wave and particle nature) by symmetry considerations matter should also exhibit dual nature.

The waves associated with material particles in motion are called **matter waves or de-Broglie waves.**

de Broglie wavelength:

The energy of a photon is E = hν………… (1)

Where **h** is the Planck's constant and 'ν ' is the frequency of radiation.

From Einstein's theory of relativity the

Energy of a photon is E = mc^2 ……………… (2)

Where m is mass of the photon and c is the speed of light in free space.

Comparing equation (1) and (2) we gethv = mc^2

As $\nu = \frac{c}{\lambda}$ the above equation becomes $\frac{\mathbf{hc}}{\boldsymbol{\lambda}} = \mathbf{mc^2}$ **or** $\boldsymbol{\lambda} = \frac{\mathbf{h}}{\mathbf{mc}}$

Where mc represents the momentum of the photon denoted by p.

Thus $\boldsymbol{\lambda} = \frac{\mathbf{h}}{\mathbf{p}}$ …………………………..…… (3)

de – Broglie suggested that the above expression can be applied even to material particles. Thus the expression for de – Broglie wavelength can be written as:

$\boldsymbol{\lambda} = \frac{\mathbf{h}}{\mathbf{mv}}$ where p = mv with m being the mass and v the velocity of particle.

Note: 1) For a given velocity, larger the mass m, shorter is λ. Therefore it is not easy to measure experimentally the wavelength of the heavy particles.

(2) Davisson and Germer in 1927, by diffraction method, detected the wave associated with electrons.

The discovery of matter waves led to new field called quantum mechanics.

(3) The kinetic energy of a particle is given by $K = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$ where p= mv

Thus the de – Broglie wavelength $\boldsymbol{\lambda} = \frac{\mathbf{h}}{\sqrt{\mathbf{2mK}}}$

(4) When an electron is accelerated with a potential difference of V volts then

$$\mathbf{eV = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} or p = \sqrt{2meV}.} \quad \mathbf{Thus}\boldsymbol{\lambda} = \frac{\mathbf{h}}{\sqrt{\mathbf{2meV}}}$$

For an electron e = 1.6 x 10^{-19} C, m = 9.1 x 10^{-31} kg and h = 6.625 x 10^{-34} Js

Therefore $\boxed{\boldsymbol{\lambda} = \frac{\mathbf{12.27}}{\sqrt{\mathbf{v}}}\mathbf{A^0}}$

(5) de-Broglie wavelength of a particle of mass m at temperature T K is given by

$$\boldsymbol{\lambda} = \frac{\mathbf{h}}{\sqrt{\mathbf{3mkT}}} \qquad \mathbf{or} \qquad \boldsymbol{\lambda\alpha}\frac{\mathbf{1}}{\sqrt{\mathbf{T}}}$$

Properties of matter waves:

- Matter waves are not electromagnetic.
- Wave like properties are exhibited by both charged and neutral particles.
- Larger the particle, smaller is its de – Broglie wavelength.
- The faster the particle moves: smaller is it de – Broglie wavelength.
- The de-Broglie wavelength of a, moving particle is independent of the charge and nature of the particle.
- The velocity of matter depends on the velocity of material particle.

The Velocity of the de Broglie Wave (Phase Velocity):

A de Broglie wave is associated with a moving body. Hence we expect that the wave has the same velocity as that of the body.

Let v_p be the velocity of de Broglie wave associated with a particle moving with a velocity v. The velocity of de Broglie wave is often known as **phase velocity** and is given by

$$v_p = \nu\lambda \ldots\ldots\ldots\ldots\ldots (1)$$

Where λ is the de Broglie wavelength and ν the frequency.

To find the frequency, we equate the quantum expression E = hν with the relativistic formula for total energy E = mc^2 to obtain

$$h\nu = mc^2 \quad or \quad \nu = \frac{mc^2}{h} \ldots\ldots\ldots\ldots\ldots (2)$$

The de Broglie wave velocity is therefore

$$v_p = \nu\lambda = \frac{mc^2}{h} \times \frac{h}{mv}$$

$$v_p = \frac{c^2}{v} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

This equation gives the relationship between velocity of de Broglie waves and velocity of moving particle in a medium. For a material particle, this relation has two distributing features

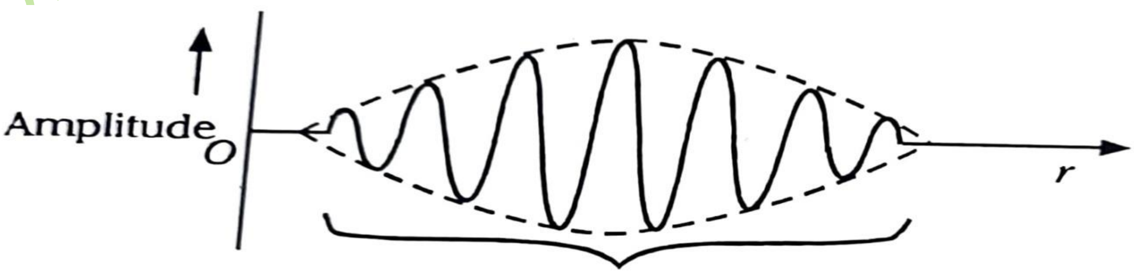
1. The velocity v_p of the de Broglie wave is not the same as the velocity of the moving particle.
2. Since the particle velocity v is always less than the velocity of light c. the velocity v_p of the de Broglie waves comes out to be greater than c which violates the fundamental postulate of the theory of relativity.

In order to understand this unexpected result, the concept of phase velocity and group velocity was introduced.

Wave and Group Velocities:

A group of waves need not have the same velocity as the waves themselves.

Schrodinger postulated that a moving particle (electron, proton etc) is equivalent to wave packet or group rather than a single wave. A packet comprises a group by waves. Each wave component propagates with a definite velocity, called the **wave velocity or phase velocity**. In a wave packet, each with slightly different velocity and wavelength, with phases and amplitudes so chosen that they interfere constructively over only a small region of space where the particle can be located. Outside the region of space the waves interfere destructively so that the amplitude reduces to zero rapidly. The amplitude of one dimensional wave-packet resembles the curve as shown in Fig.



This packet of waves which moves with its own velocity is called **group velocity**, v_g .**The velocity with which the group of wave's moves is called the group velocity** and is equal to the velocity of the particle. The velocity of the individual waves forming the wave packet is called the **phase velocity,** v_p The envelope of the wave packet moves at the group velocity while within the envelope each individual component wave moves with phase velocity.

$$v_p = \frac{\omega}{k} \ and \ \ v_g = \frac{d\omega}{dk}$$

where ω is the angular frequency and k is the wave number of de Broglie waves.

Relation between Phase and Group velocity of de Broglie Waves:

The phase velocity of the de Broglie waves is given by

$$v_p = h\nu = \frac{\omega}{k} \ \ \ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

We know that , $v_p = \frac{c^2}{v}$ $\ldots\ldots\ldots\ldots..(2)$

where v is the velocity of the electrons and c is the velocity of light in vacuum.

According to de Broglie, $v = v_g$, Hence we have

$$\boldsymbol{v = \frac{c^2}{v_g} \ \ or \ \ \ v_p \times v_g = c^2}$$

This is the relation between the phase and group velocities of the de Broglie waves

The experiments of Davisson and Germer and of G. P. Thomson on electron diffraction confirm the wave nature of matter.

G.P Thomson's experiment:

In 1928 G.P. Thomson devised the following experiment to demonstrate the wave nature of electrons.

Principle of the experiment:

Electrons accelerated to a high p.d. are made to fall on a thin gold foil. The emergent beam is made to strike a photographic plate. Distinct sharp circular rings are obtained on the plate showing that the electron beam is diffracted by the gold foil thereby proving the wave nature of electrons.

Description of the apparatus:

The experimental arrangement is as shown in (fig.1). A beam of electrons produced in a discharge tube is collimated by passing it through a fine hole A. The beam is then made to fall on thin gold foil F of thickness about 10^{-8} m. The emergent beam then strikes a photographic plate P. A high vacuum is maintained in the region between F and P.

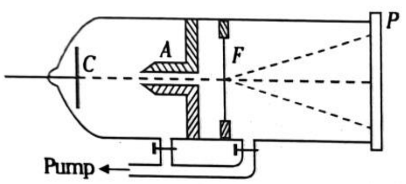


Fig. 1

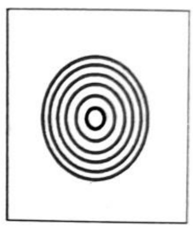


Fig. 2

Procedure:

Electrons from the cathode accelerated through a known large p.d. (50,000 V) are collimated by passing them through a fine hole in A. After striking the gold foil F the beam falls on the photographic plate P. When the plate is developed a symmetrical pattern consisting of a set of concentric rings with a spot at the centre is obtained (Fig.2).

If the electrons behave like particles they are scattered by the gold foil and will produce a patch on the photographic plate. But a set of sharp circular rings is obtained. This pattern is similar to that obtained in the Debye Scherer powder method for the analysis of crystals using X-rays. The formation of the pattern is explained as follows:

The gold foil consists of a large number of tiny crystals with random orientation. Those crystals which are at the required angle (defined by Bragg's law) scatter the incident electron beam. Circular rings are produced due to the intersection of the cones of diffracted electrons with the photographic plate. To ensure that the pattern is due to the diffracted electrons and not due to secondary X-rays generated by the electrons in passing through the foil, the electron beam is deflected by the application of a magnetic field. The entire pattern shifts showing that the pattern is not produced by X-rays.

The experiment clearly proves that electrons behave as waves since a diffraction pattern can be produced only by waves and not by particles.

Verification of de-Broglie equation

G.P. Thomson calculated the wavelength of the electrons using the equation

$\lambda = \frac{12.27A^0}{\sqrt{V}}$ V being the p.d. in volt through which the electron beam is accelerated.

Calculation of λ from the pattern: Let PQ be the incident beam passing through the film at Q and incident on the crystal plane at the glancing angle as shown in Fig(3). The beam reflected by the crystal at Q strikes the photographic plate at R.

Let OR =r and QO = D ,Let RQO=2θ

By Bragg's law, 2d sin θ = nλ ...(1)

From triangle QRO, tan 2θ = r/D

Since θ is small tan 2θ = 2θ and sin = θ≈ θ

$$\therefore \ \theta = \frac{r}{2D} \ \ldots\ldots\ldots\ldots (2)$$

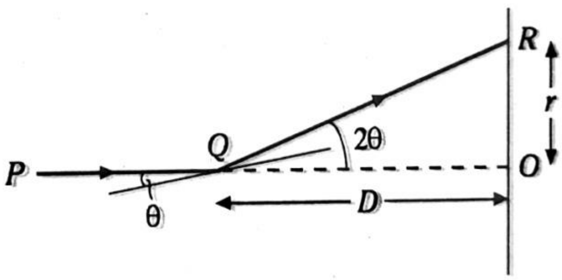


Fig. 3

and (1) becomes 2d θ = nλ or

$$\theta = \frac{n\lambda}{2d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

From eqns (2) and (3), $\frac{n\lambda}{2d} = \frac{r}{2D}$ $\therefore \boldsymbol{\lambda} = \frac{\boldsymbol{rd}}{\boldsymbol{nD}}$

The value of λ calculated from this equation agrees with the value calculated from equation (1). This proves the wave nature of electrons.