KARNATAK UNIVERSITY B.Sc III Sem NEP Unit –I WAVE MOTION

Wave Motion: Types of waves, Plane and spherical waves, Transverse and longitudinal wave. Displacement, velocity and pressure curve. Expression for a plane progressive wave, particle velocity. **Relation between particle velocity and wave velocity**. Differential equation of wave motion, mention of differential equation of three dimensional waves. Derivation of energy density of a plane progressive wave. Distribution of energy in a plane progressive wave. Expression of intensity of progressive wave. Superposition of waves: Interference-Beats, theory of beats (analytical treatment). Super position of two perpendicular SHM: Lissajous figures with equal and unequal frequency- analytical treatment and use of Lissajous figures. Velocity of transverse wave along stretched string, wave equation for transverse wave in a string. Longitudinal (sound) waves in fluid medium -derivation of Newton's formula - Laplace's corrections for Newton's formula. Effect of pressure, temperature and humidity on the velocity of sound. Group velocity-its relationship with wave (or phase) velocity. Concept of resonance. Theory of Helmholtz resonator.

WAVE MOTION

A wave is a sort of disturbance which is transmitted in a medium without the bulk movement of particles of the medium. Or A wave is a sort of disturbance when a group of particles of the medium are disturbed, the pattern of disturbance that travels through the medium due to the periodic motion of the particles of the medium about their equilibrium position, with the transfer of energy and momentum and without the transfer of matter (particles) is called a **wave.** **OR** The Disturbance set up in a medium is called **wave**. The propagation of disturbance is called **wave motion.** A number of waves can be noticed around us. Their sources and properties are different. But they are all carriers of energy. Some of the commonly encountered waves are as follows.

i) Ripples on the surface of water. ii) Light waves iii) Sound waves iv) Radio waves
v) Micro waves ,and vi) Seismic waves.

CLASSIFICATION OF WAVES:

I. Based on need of medium waves are classified in to two groups:

i) Mechanical waves and ii) Non-mechanical waves.

Waves which requires material medium for their propagation are called **mechanical waves.**

Eg : Waves on the surface of water, sound waves ,seismic waves and waves on stretched string.

Waves which do not requires material medium for their propagation are called **non- mechanical waves.** These waves propagate through vacuum as well as through matter. They are also called **electromagnetic waves.** Eg: Light waves ,radio waves , heat waves and television signals.

Mechanical wave	Non mechanical wave
➢ Requires a material medium for their propagation.	➢ Do not require a material medium for their propagation.

➢ Particles of the medium oscillate ➢ Can be longitudinal (or) transverse in nature ➢ Travels at relatively lower speed in medium ➢ Doppler effect is asymmetric Eg : Sound waves	➢ Electric and magnetic field oscillate. ➢ Are always transverse in nature. ➢ Travels at relatively higher speed in medium ➢ Doppler effect is symmetric Eg: Light waves.

II Based on mode of propagation waves are classified in to three groups:

a) One dimensional b) two dimensional and c) three dimensional waves.

Waves travelling along a straight line are <u>**one dimensional**</u>. Eg : Waves on a stretched string.

Waves travelling in a plane are <u>**two dimensional**</u>**.** Eg : Ripples on the surface of water.

Waves travelling in space are <u>**three dimensional**</u>. Eg : Sound waves and light waves.

III. Based on vibrations of particles waves are classified into two groups:

1) Transverse waves and 2) Longitudinal waves.

1. **Transverse wave:** If the particles of the medium vibrates perpendicular to the direction of Propagation of the wave, then the wave is called **transverse waves.** Ex: light waves.

2. **Longitudinal waves:** If the particles of the medium vibrate along the direction of propagation of the Wave, then the wave is called **longitudinal waves**. Ex: sound waves.

In transverse waves, alternatives crest & trough are formed. In longitudinal waves alternative compression & rarefaction is formed.

THE DIFFERENCES BETWEEN LONGITUDINAL AND TRANSVERSE WAVES

Longitudinal wave	**Transverse wave**
• The particles of the medium oscillate along (parallel to) the direction of the propagation of the wave. • These waves travels in alternate compression & rarefaction (compression are the regions of higher density & rarefaction are the regions of lower density) • They are always mechanical wave • The pressure and density varies as the wave propagates.	• The particles of the medium (electric & magnetic fields) oscillate at right angles (perpendicular) to the direction of propagation of the wave. • Crests and troughs. (Crests are highly raised portions of a wave & troughs are highly depressed portion of a wave). • They are either mechanical (or) non mechanical wave. • The pressure and density do not vary as the wave prorogates.

• They can travel in solids, liquids and gases. • These waves cannot be polarized. • Velocity of a longitudinal wave in a gas is given by $v = \sqrt{\frac{B}{\rho}}$ where B is the bulk modulus and ρ is the density Eg: Sound waves.	• They can travel in solids and on the gases. surface of liquids, if the waves are mechanical • These waves can be polarized. • Velocity of a transverse wave on a stretched string is given by $v = \sqrt{\frac{T}{\mu}}$ where T is the tension and μ is linear density. Eg : Waves on a string, Light waves.

Plane wave: It is an undamped wave since its amplitude A is constant along the direction of propagation.

Spherical waves: Spherical waves are those for which the wave front is the surface of a sphere. They emanate from a point source in the 3-D space (light wave is spherical wave).

Definition of some important terms

1. **Amplitude: (A**) Maximum displacement of the particle from its mean position is called **amplitude of the wave.**
2. **Period (T)** :- The time during which one complete wave is set up in a medium is called **period**. Unit is second.
3. **Frequency (f):-** Number of waves set up in a medium in one second is called **frequency.**

   $\boldsymbol{f = \frac{1}{T}}$ Its unit is per second or cycles per second or hertz.
4. **Wavelength (λ)** : Distance between two consecutive particles which are in a same state of vibration or in same phase. In case of transverse waves, wavelength is the distance between two consecutive crests or troughs. In longitudinal waves, it is the distance between two consecutive compression or rarefactions.
5. **Wave velocity:** Distance traveled by the wave in one second is called **wave velocity**.

6.**Phase:** Phase of a vibrating particle at any instant indicates the state of vibration of the particle at that instant. It is given by fraction of time period that has been elapsed since the particle last passed through its mean position in the positive direction.

Derive v =fλ

Consider a progressive wave of wavelength (λ), frequency (f) , A be its amplitude, T is its period and is moving with a wave velocity (v).

$$\boldsymbol{Wave\ velocity = \frac{\text{Distance traveled}}{\text{Time taken}} = \frac{\text{wavelength}}{\text{Period}} = \frac{\lambda}{T}}$$

$$\boldsymbol{v = \frac{\lambda}{T} \quad but\ \frac{1}{T} = f \quad \therefore v = f\lambda}$$

Relation between phase difference and path difference: If two particles are separated by a distance equal t wavelength $\boldsymbol{\lambda}$ of the wave propagating through the medium, the phase difference between them is $\boldsymbol{2\pi}$ radian. I two particles are separated by a distance $\boldsymbol{\Delta x}$**,** the phase difference $\boldsymbol{\Delta\Phi}$ between them is given by $\boldsymbol{\Delta\Phi} = \frac{2\pi}{\lambda}.\Delta x$

Progressive wave: The wave which travels continuously in a medium in the same direction with a constan amplitude is called a **progressive wave**.

The simplest form of a progressive wave is a sinusoidal or simple harmonic. Due to wave motion, the particles of the medium execute S.H.M. about their mean position. Hence the name simple harmonic wave.

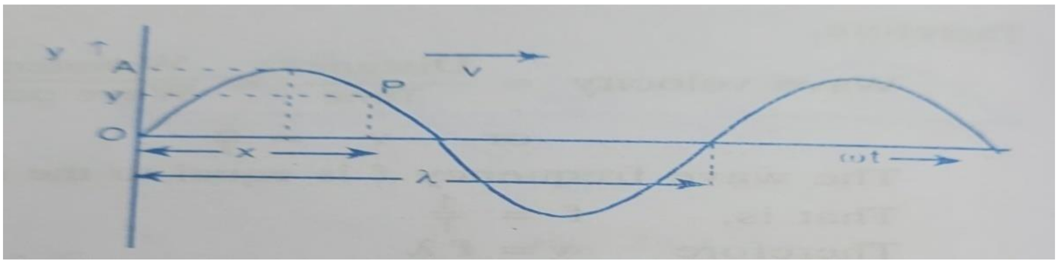
Characteristics of wave motion

1. Wave motion is the disturbance produced in the medium by the repeated periodic motion of particles of the medium.
2. Only the wave travels forward whereas the particles of the medium vibrate about their mean positions.
3. There is a regular phase change between the various particles of the medium. The particle ahead starts vibrating a little later than a particle just preceding it.
4. The velocity of the wave is different from the velocity with which the particles of the medium are vibrating about their mean positions. The wave travels with a uniform velocity whereas the velocity of the particles is different at different positions. It is maximum at the mean position and zero at the extreme position of the particles.

Characteristics of Progressive waves:

- The disturbance produced at any point in a medium is propagated by the repeated periodic motions of the particles of the medium.
- The propagation of disturbance is due to the elastic and inertial property of the medium.
- There is a transfer of energy from one point to another in the form of disturbance.
- In a homogeneous medium the wave propagates with a constant velocity called wave velocity however particle velocity is different at different instant.
- Every particle of the medium vibrate about their mean position with same amplitude & frequency However the phase of different particle is different at a given instant
- The waves can undergo reflection, refraction, interference and diffraction.
- Transverse waves undergo polarization. Longitudinal waves do not undergo polarization.
- No particle in the medium is permanently at rest.

Equation for a progressive wave:



Consider a progressive wave propagation along positive x-axis with a velocity 'v'. Consider a particle at origin O in a given medium vibrating simple harmonically, its displacement y at any instant of time 't' is given by

y= A sin ωt …………………..(1)

Where A → amplitude of vibration, ω→ angular frequency ω= 2πf = $\frac{2\pi}{T}$, T being the period of the wave and f is the frequency .

Now consider another particle of the medium at the point P at a distance x from origin O. The wave starts from O would reach the point P in a time x/v. Therefore the displacement of particle at P at a time't' will be same as that of particle at O at a time (t- $\frac{x}{v}$) and it is given by

$$\boldsymbol{y = A\sin\omega\left(t - \frac{x}{v}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Equation (2) represents the displacement of particle at P at a time't'. For different values of x and t equation (2) represents the state of motion different particles in the medium at different instant of time. Hence equation (2) represents equation of a simple harmonic wave travelling along positive x-axis. Equation (2) can also be written in following form.

$$\boldsymbol{since\ \omega = 2\pi f = \frac{2\pi}{T} \quad and\ v = f\lambda = \frac{\lambda}{T}}$$

$$\boldsymbol{y = A\sin\frac{2\pi}{T}\left(t - \frac{x}{v}\right)} \ldots\ldots\ldots\ldots (3)$$

$$\boldsymbol{y = A\sin\frac{2\pi}{\lambda}(vt - x)} \ldots\ldots\ldots\ldots (4)$$

$$\boldsymbol{y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(5)$$

$$\boldsymbol{y = A\sin}(\omega t - kx) \ldots\ldots\ldots.(6)$$

Where ω = $\frac{2\pi}{T}$ and k= $\frac{2\pi}{\lambda}$ and k is called a **propagation constant** or angular wave number. Equations 3, 4, 5 and 6 also represents equations of a progressive wave.

Note: If wave is propagating along –ve x-axis then $\boldsymbol{y = A\sin\omega\left(t + \frac{x}{v}\right)}$

Generally, **y = Asin (ωt + kx)**

Particle velocity and wave velocity

The equation of progressive wave is given by

$$y = Asin\,\frac{2\pi}{\lambda}(\mathrm{vt-x}) \ldots\ldots\ldots..(1)$$

The particle velocity is defined as the rate of change of displacement y with respect t Hence differentiating equation (1) w.r.t 't' we have

$$\frac{dy}{dt} = \frac{2\pi Av}{\lambda} Cos\,\frac{2\pi}{\lambda}(\mathrm{vt-x}) \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

The maximum value of particle velocity is $\left(\frac{dy}{dt}\right) max = \frac{2\pi Av}{\lambda}$ $\quad$ $when\ Cos\,\frac{2\pi}{\lambda}(\mathrm{vt-x}) = 1$

$$\textbf{Maximum particle velocity} = \frac{2\pi A}{\lambda} \times wave\ velocity$$

Differentiating equation (1) w.r.t ‘ x’ we have

$$\frac{dy}{dx} = -\,\frac{2\pi A}{\lambda} Cos\,\frac{2\pi}{\lambda}(\mathrm{vt-x}) \ldots\ldots\ldots\ldots\ldots(3)$$

$\frac{dy}{dx}$ represents the strain or compression.

When $\frac{dy}{dx}$ is positive a rarefaction takes place and $\frac{dy}{dx}$ is negative a compression takes place

Comparing equation (2) and (3) we get particle velocity $\frac{dy}{dt} = -v\,\frac{dy}{dx} \ldots\ldots\ldots.(4)$

Particle velocity = Wave velocity x Slope of the displacement curve

Differential equation of wave motion:

Consider a progressive wave propagation along positive x-axis with a velocity ‘v’. Consider a particle at origin O in a given medium vibrating simple harmonically, Now consider another particle of the medium at the point P at a distance x from origin O. its displacement y at any instant of time ‘t’ is given by

$$y = A\,sin(\omega t - kx) \ldots\ldots\ldots\ldots.(1)$$

Where $\omega = \frac{2\pi}{T} = 2\pi\nu$ and k= $\frac{2\pi}{\lambda}$ and k is called a **propagation constant** or angular wave number

Differentiating equation (1) w.r.t ‘x’

$$\frac{dy}{dx} = \frac{d}{dx}[A\,sin(\omega t - kx)]$$

$$\frac{dy}{dx} = -Ak\,cos(\omega t - kx)\text{---------------}(2)$$

Differentiating equation (2) w.r.t ‘x’

$$\frac{d^2y}{dx^2} = -Ak(-k)[-\,sin(\omega t - kx)]$$

$$\frac{d^2y}{dx^2} = -Ak^2 sin(\omega t - kx) - - - - - - - - -(3)$$

Differentiating equation (1) w.r.t ‘t’ keeping x constant

$$\frac{dy}{dt} = \frac{d}{dt}[A\,sin(\omega t - kx)]$$

$$\frac{dy}{dx} = A\,\omega\,cos(\omega t - kx)\text{---------------}(4)$$

Differentiating equation (4) w.r.t 't' keeping x constant

$$\frac{d^2y}{dt^2} = A\omega.\omega[-\sin(\omega t - kx)]$$

$$\frac{d^2y}{dt^2} = -A\omega^2 \sin(\omega t - kx) \ldots\ldots..(5)$$

Divide equation (5) by equation (3) we get

$$\frac{\frac{d^2y}{dt^2}}{\frac{d^2y}{dx^2}} = \frac{-A\omega^2 \sin(\omega t - kx)}{-Ak^2 \sin(\omega t - kx)} = \frac{\omega^2}{k^2}$$

But $\frac{\omega}{k} = \frac{2\pi\nu}{\frac{2\pi}{\lambda}} = \nu\lambda\ by\ defination = \nu\lambda = v\ the\ velocity\ of\ the\ wave$

$$\therefore \frac{\frac{d^2y}{dt^2}}{\frac{d^2y}{dx^2}} = \frac{\omega^2}{k^2} = v^2 \quad or \quad \frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \ldots\ldots\ldots(6)$$

Equation (6) is the differential equation of the wave motion

Mention of differential equation of three dimensional wave:

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} + \frac{d^2y}{dy^2} + \frac{d^2y}{dz^2} \quad OR \quad \frac{d^2y}{dx^2} + \frac{d^2y}{dy^2} + \frac{d^2y}{dz^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

Intensity of the wave: The amount of wave energy transferred per second per unit area normal to the direction of propagation of the wave is called intensity of the wave.

It is also defined as power transmitted per unit area held normal to the direction of propagation of the wave. **$I = 2\pi^2 f^2 A^2\rho v$**

Where, f = frequency of the wave. A = Amplitude of the wave ρ = density of the medium v = velocity of the wave

The SI unit of intensity of wave is Wm^{-2}

Energy per unit volume = energy density = I/v = **$2\pi^2 f^2 A^2\rho$**

Superposition of waves:

When two waves of same type meet simultaneously over a common region in a medium, they are said to be superimposed or superposed. This is known as superposition of wave.

The principal of superposition of wave states that when two or more waves of same kind arrive at a point simultaneously are superposed then their resultant displacement of any particle in a medium is equal to vector sun of their individual displacements provided they all small.

If y_1, y_2, y_3 etc are the displacements of individual waves then resultant displacement

$y = y_1 + y_2 + y_3 + \ldots\ldots.$ etc.

Consider two progressive waves of same frequency, same wavelength travelling in the same direction in a medium. The waves may be represented as

$$Y_1 = A_1 \sin(\omega t - kx)\ \ and\ \ Y_2 = A_2 \sin[(\omega t - kx) + \phi)]$$

Where A_1 and A_2 are the amplitudes of the waves and ϕ is the phase difference between them. When these waves superpose,the resultant displacement at any point is given by

$$y = y_1 + y_2 = A_1 \sin(\omega t - kx) + A_2 \sin[(\omega t - kx + \phi)]$$
$$= A \sin[(\omega t - kx + \alpha)]$$

Where A is the resultant amplitude, given by $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$ and phase difference α is given by $\tan\alpha = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$

when $\phi = 0, 2\pi, 4\pi, \ldots\ldots\ldots 2n\pi$ the resultant amplitude is maximum and is given by $A_{max} = A_1 + A_2$

when $\phi = \pi, 3\pi, 5\pi, \ldots\ldots\ldots (2n+1)\pi$ the resultant amplitude is minimum and is given by $A_{min} = A_1 - A_2$

Following are the important phenomena explained based on the principal of superposition.

- **Beats:** Two waves of slightly different frequencies travelling in the same direction when they superposed, **beats** are produced.
- **Interference:** If two waves of same frequencies travelling in the same direction when they superposed, **interference** is observed.
- **Stationary waves:** If two waves of same frequency travelling in opposite direction when they super imposed, stationary wave are formed.

BEATS:- If two source of sound of equal frequencies are sound together, a single sound is heard whose loudness steadily decreases to zero. On the other hand if two source of sound of nearly equal frequencies are sounded together, the loudness of the sound produced varies between maximum (**waxing)** and minimum **(waning)** periodically with time. **This phenomenon of waxing and waning of sound periodically with time is called beats.** The time interval between two consecutive waxing or waning is called **beat period**. The number of beat hard per second is called **beat frequency**. In other words, the beat frequency is the number waxing or waning hard per second it can be shown that the beat frequency is equal to the difference

in the frequency of sources of sound producing beats. Thus beat frequency = ν1~ν2. Where ν_1 and ν_2 are frequencies of sources of sound producing beats.

Analytical treatment of beats:

Consider two sound waves of same amplitude A and slightly different frequencies ν1 & ν2 travelling in the same direction through air. The displacements of a particular particle in the medium at a time t due the two waves is

$$\mathbf{y_1 = A \sin\omega_1 t \quad \text{where } \omega_1 = 2\pi\nu_1 \qquad y_2 = A \sin\omega_2 t \quad \text{where } \omega_2 = 2\pi\nu_2}$$

The resultant displacement y of the particle due to superposition of these two waves is

$$y = y_1 + y_2 = A\sin\omega_1 t + A\sin\omega_2 t$$
$$y = A\{\sin\omega_1 t + \sin\omega_2 t\}$$

$y = A\left\{2\sin\left(\frac{\omega_1 t-\omega_2 t}{2}\right)\cos\left(\frac{\omega_1 t-\omega_2 t}{2}\right)\right\}$ since $\sin C + \sin D = 2\sin\left[\frac{C+D}{2}\right]\cos\left[\frac{C-D}{2}\right]$

$$y = A\left\{2\sin\left(\frac{2\pi\nu_1 t-2\pi\nu_2 t}{2}\right)\cos\left(\frac{2\pi\nu_1 t-2\pi\nu_2 t}{2}\right)\right\}$$

$$= \left\{2A\cos 2\pi\left(\frac{\nu_1-\nu_2}{2}\right)t\right\}\left\{\sin 2\pi\left(\frac{\nu_1+\nu_2}{2}\right)t\right\}$$

$$= R\sin 2\pi\left(\frac{\nu_1-\nu_2}{2}\right)t \quad \ldots\ldots\ldots\ldots\ \ldots\ldots(1)$$

Where $R = 2A\cos 2\pi\left(\frac{\nu_1-\nu_2}{2}\right)t$ ………………………..(2) is the amplitude of the resultant wave

From equations (1) and (2) it is clear that

The resultant wave is a harmonic wave of amplitude R and frequency $\left(\frac{\nu_1+\nu_2}{2}\right)$ which is the average of the frequencies of the two waves.

The amplitude R of the resultant wave varies periodically but slowly with a frequency ($\frac{\nu_1-\nu_2}{2}$)

The resultant s amplitude is maximum when

$$\cos 2\pi\left[\left[\left(\frac{\nu_1-\nu_2}{2}\right)\right]\right]t = \cos\pi(\nu_1-\nu_2)t = \pm 1$$

ie., $\pi(\nu_1-\nu_2)t = n\pi$ or $t = \frac{n}{\nu_1-\nu_2}$ ie when $t = 0, \frac{1}{(\nu_1-\nu_2)}, \frac{2}{(\nu_1-\nu_2)}, \frac{3}{(\nu_1-\nu_2)}$ ………………

The time interval between two successive maximum is equal to $\frac{1}{(\nu_1-\nu_2)}$

The resultant s amplitude is minimum when

$$\cos 2\pi\left[\left[\left(\frac{\nu_1-\nu_2}{2}\right)\right]\right]t = \cos\pi(\nu_1-\nu_2)t = 0$$

ie., $\pi(\nu_1-\nu_2)t = (2n+1)\frac{\pi}{2}$ or $t = \frac{(2n+1)}{2(\nu_1-\nu_2)}$ ie when $t = 0, \frac{1}{2(\nu_1-\nu_2)}, \frac{3}{2(\nu_1-\nu_2)}, \frac{5}{2(\nu_1-\nu_2)}$

The time interval between two successive minimum is equal to $\frac{1}{(\nu_1-\nu_2)}$

The beat period is the time required for one beat to be formed or the time interval between two consecutive maximum (waxing) or minimum (waning) is called **beat period**.

Thus, beat period $T_b = \frac{1}{(\nu_1-\nu_2)}$

Hence beat frequency $= \nu_b = \frac{1}{T_b} = (\nu_1-\nu_2)$

Hence beat frequency is the difference in frequencies of the component waves.

APPLICATIONS OF BEATS: -1) Determination of unknown frequency2) Tuning of musical instruments

3) Detection of harmful gases in mines

NOTE: 1) **Loading** of one of the prongs of a turning fork always decreases the frequency of the fork.

2) **Filling** of one of the prongs always increases the frequency of the fork.

Speed of Transverse Wave on a Stretched String:

The speed of a mechanical wave is determined by the inertial and elastic properties of the medium. When a transverse wave propagates along a stretched string, the restoring force is provided by the tension T in the string. The inertial property is represented by linear mass density m (mass per unit length) of the string.

The speed of transverse wave in a string can be obtained by dimensional analysis. Since tension T is a force, its dimensional formula is $[MLT^{-2}]$. The dimensional formula for linear mass density of mass per unit length m is $[ML^{-1}]$. We get

$$\frac{[MLT^{-2}]}{[ML^{-1}]} = [L^2T^{-2}] = [LT^{-1}]^2$$

But $[LT^{-1}]$ is the dimensional formula for speed v. Hence we can write

$$v^2 \alpha \ \frac{T}{m} \quad or \ \ v \ \alpha \ \sqrt{\frac{T}{m}}$$

$\therefore v = C\sqrt{\frac{T}{m}}$ where C is constant.

The exact derivation shows that C = 1.

The speed of transverse waves on a stretched string is given by $v = \sqrt{\frac{T}{m}}$

Thus, the speed depends on tension T of the stretched string and its linear mass density m.

SPEED OF SOUND IN GASES:- In a given media sound travels as a longitudinal wave. Its velocity of propagation is determined by the elastic and inertial properties of the medium.

NEWTON'S FORMULA:- Newton showed that the velocity of longitudinal wave in a medium is given by

$$\boldsymbol{v} = \sqrt{\frac{\boldsymbol{E}}{\boldsymbol{\rho}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where 'E ' is modulus of elasticity and ρ is the density of the medium,

For solids , E = Y (Young's modulus) and for a gaseous medium E = B (bulk modulus).

$$\therefore \boldsymbol{v} = \sqrt{\frac{\boldsymbol{B}}{\boldsymbol{\rho}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The propagation of sound waves in a gaseous medium is accompanied by changes in pressure and volume throughout the medium. Newton assumed that these changes take place at isothermal conditions. Under this condition, the bulk modules of a perfect gas is given by

equal to the pressure 'P' .

$$\boldsymbol{v} = \sqrt{\frac{\boldsymbol{P}}{\boldsymbol{\rho}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\,(\mathbf{3})$$

Equation (3) is called Newton's formula for velocity of sound. Taking normal pressure $\mathbf{P = 1.013 \times 10^5}$ $\mathbf{Nm^{-2}}$ (Pascal) and density of air at N.T.P. as $\mathbf{1.293\ kgm^{-3}}$ the Newton's formula gives the velocity of sound in air at NTP is to be about 280 ms^{-1}. This is much less than the experimental value which is about 332 ms^{-1}. Thus the value of velocity of sound obtained from Newton's formula does not agree with the experimental value. The theoretical value is about 16 % less than the experimental value. This large error cannot be explained. Hence Newton's formula requires a correction and the required correction was given by Laplace.

NEWTON'S – LAPLACE FORMULA:- According to Laplace, Since air is an insulator, isothermal condition can not prevail. As a result, changes in pressure and volume are brought about under adiabatic conditions. For a adiabatic changes it can be shown that the bulk modulus 'B' is equal to 'γp' where 'γ 'is the ratio of two specific heats. for adiabatic changes is **PV^{γ} = constant.** Where $\gamma = \frac{C_p}{C_v}$

$$v = \frac{\gamma P}{\rho} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(4)$$

This formula is known as Newton's- Laplace formula for velocity of sound in a gas.

For air γ =1.41 using this value in equation (4). The velocity of sound in air at N.T.P. is found to be nearly 332.4 ms^{-1} which is in good agreement with the experimental value.

FACTORS AFFECTING VELOCITY OF SOUND IN GASES:-

1) EFFECT OF PRESSURE:- According to Boyle's law constant temperature, PV = constant. Where 'P' is the pressure and 'V' is the volume of mass m of air. If 'ρ' is the density of air at pressure P

Volume = $\frac{\text{mass}}{\text{density}}$ = V = $\frac{m}{\rho}$ ⇒ PV = $\frac{Pm}{\rho}$ = a constant

Since mass 'm' remains constant $\frac{P}{\rho}$ also remains constant ∴ $v = \sqrt{\frac{\gamma P}{\rho}}$ = a cost since γ is constant for air.

Thus the velocity of sound in air is independent of pressure provided temperature remains constant.

2) EFFECT OF TEMPERATURE:- At constant pressure, as temperature increases, the density of gas decreases. Hence , speed of sound in a gas increases with temperature.

For one mole of a gas, $\rho=\frac{M}{V}$ where M is molecular mass and V is the volume.

$$\therefore v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{M}} = \sqrt{\frac{\gamma RT}{M}}$$

Thus $v \propto \sqrt{T}$

Thus velocity of sound in air is directly proportional to the square root of absolute temperature.

Let v_1 and v_2 are velocity of sound in air at temperature T_1 and T_2 respectively.

Then, $v_1 = \sqrt{\frac{\gamma RT_1}{M}}$ and $v_2 = \sqrt{\frac{\gamma RT_2}{M}}$ ∴ $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

Effect of Humidity: The density of water vapour-at NTP = 0.8 kg m^{-3}whereas the density of dry air at NTP = 1.293 kg m^{-3}. Therefore, water vapour has a density less than the density of dry air. Consequently the density of moist air is less than the density of dry air. As the velocity of sound is more in a medium of lesser density, velocity of sound in moist air is more than the velocity of sound in dry air.