UNIT-3 MAGNETOSTATICS

Syllabus: *Overview of basics of Magnetostatics, Statement of Biot-Savart' law, derive the expression for magnetic field due to Straight conductor carrying current, mention the expression for the field along the axis of a circular coil and discuss the special cases. Tangent law, Helmholtz galvanometer-principle, construction and working. Ampere's circuital law-statement, proof and its applications (for D. C.) to derive the magnetic field due to Solenoid and Toroid.*

INTRODUCTION:

Earlier electricity and magnetism were treated as separate subjects. Electricity dealt with interactions of charged bodies while magnetism dealt with interactions of magnets and compass needles. However, in 1820, it was realized that they were intimately related. Experiments of Danish scientist H.C Oersted showed that a compass needle deflected by passing electric current through a wire near the needle. From these experiments, Oersted concluded that electric current through a conductor produces magnetic field in the surrounding space. Ampere supported this observation by saying that electric charges in motion produce magnetic fields. Michael Faraday showed that moving magnets or changing magnetic fields generate electricity. Maxwell unified the laws of electricity and magnetism and developed a new field called ***electromagnetism***. Most of the phenomenon occurring around us can be described under electromagnetism.

AMPERE'S SWIMMING RULE:

Imagine a man swimming along the wire in the direction of current with his face always turned towards the needle such that current enters his feet and leaves at his head. Then, the north pole (N-pole) of the magnetic needle will be deflected towards his left hand.

MAGNETIC FORCE ON A MOVING CHARGE, LORENZ FORCE

Consider a positively charged particle q moving with a velocity v in a magnetic field of strength B as shows in the diagram. The force experienced by the charge is given by

i. The magnitude of the force $\vec{F}$ is directly proportional to strength of the magnetic field applied i.e. $F \propto B$

ii. The magnitude of the force is directly proportional to the magnitude of the charge i.e. $F \propto q$ and

iii. The magnitude of the force is directly proportional to component of the velocity in the direction perpendicular to the direction of field, i.e. $F \propto sin\theta$

$\vec{F}\ \alpha\ qvBsin\theta$ Or $\vec{F} = kqvBsin\theta$ where K is constant of proportionality and k= 1

$\therefore \vec{F} = qvBsin\theta$ ……………………….(1)

Equation (1) can be written in vector form as

$$\vec{\mathbf{F}} = \mathbf{q}\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right) \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (\mathbf{2})$$

Special Cases:

- Consider F = qvB sinθ. If v = 0, F = 0 i.e., a charged particle at rest in a magnetic field experiences no force i.e. magnetic field do not interact with stationary charge.
- If θ= 0° or 180°, **F = 0** i.e., a charged particle moving parallel or anti parallel to the direction of the field experiences no force.
- If θ = 90°, F will be maximum and its value is given by $\mathbf{F_{max} = qvB}$
- When q=0, F=0, hence the force on neutral particle is zero

Note:

➢ An electric charge q moving with a velocity v in a region having magnetic field B and electric field E will experience a resultant force $\vec{F} = q\vec{E} + q(\vec{v}X\vec{B}) = q(\vec{E} + \vec{v}X\vec{B})$ ---------------- (3)

This relation is called **Lorentz relation** and the resultant force is called the **Lorentz force.**

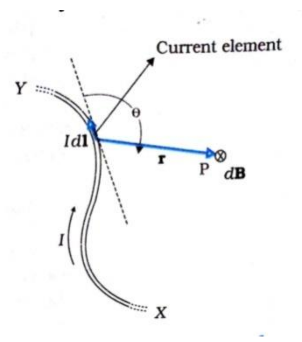
➢ As the force acts along a direction at right angles to direction of motion of the charge, the work done is zero. Thus there is no change in magnitude of velocity of the charge but only It is maximum when the conductor is perpendicular to the magnetic field (i.e. θ = 90^0)

This maximum value of the force is F = I *l* B.

BIOT-SAVARTS LAW:

Biot- Savart's law (or Laplace's law) helps to calculate the magnetic field dB at a point outside the conductor due to a small length d*l* of the conductor carrying current I. The law is stated as follows:

The magnetic field dB at a point due to a current element Idl *is*



- Directly proportional to the strength of the electric current I
- Directly proportional to the length *dl* of the current element,
- Directly proportional to the sine of the angle between the current element and line joining the point of observation with the current element i.e, $sin\theta$ and
- Inversely proportional to the square of the distance r of the point from the current element.

Consider a conductor XY through which the current is I. Let AB be a small element of a length dl. Let P be a point at a distance r from the centre O of the element and **θ** be the angle between the current element and the line OP. If dB is the magnetic field at P due to the current element AB, then

$dB \propto \frac{I\,dl\,sin\theta}{r^2}$ or $dB = K\frac{I\,dl\,sin\theta}{r^2}$ Where K is a constant.

In SI units, $K = (\frac{\mu_0}{4\pi})$ Where $\mu_o = 4\pi \times 10^{-7}$ TmA^{-1} is called absolute permeability of free space or vacuum.

$$\therefore dB = \left(\frac{\mu_0}{4\pi}\right)\frac{I\,dl\,sin\theta}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

In vector form, Biot – Savart's law is written as

$$\vec{\mathbf{dB}} = \left(\frac{\mu_0}{4\pi}\right)\frac{\boldsymbol{I\,\vec{dl}\times\hat{r}}}{\boldsymbol{r^3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The direction of dB is perpendicular to the plane containing dl and r (r is called displacement vector along OP). When θ=0, we get dB=0,

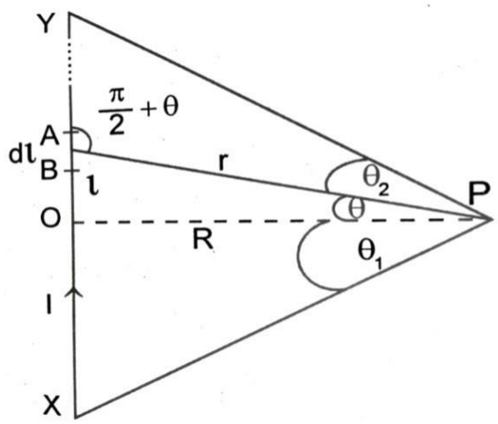
Thus the magnetic field at any point on a thin current carrying conductor itself is zero.

Magnetic field at a point due to an infinitely long straight conductor carrying current:

Consider a conductor carrying current I. let us consider a small element AB of length dl and join its mid point to the point P where a magnetic field is to be determined. Let r be the distance of the element from the point P and 'R' is the perpendicular distance from the conductor.

According to Biot –Savart's law, the magnetic field at point P due to current carrying element 'dl' is given by

$$\boldsymbol{dB} = \left(\frac{\mu_0}{4\pi}\right)\frac{\boldsymbol{Idl\,\sin(\frac{\pi}{2}+\theta)}}{\boldsymbol{r^2}} = \left(\frac{\mu_0}{4\pi}\right)\frac{\boldsymbol{Idl\ \cos\theta}}{\boldsymbol{r^2}} \quad \ldots\ldots\ldots \ \mathbf{(1)}$$



In a right angled triangle OPQ, $\tan\theta = \frac{l}{R} \Rightarrow l = R\,tan\theta$

Differentiating $dl = R\,sec^2\,\theta\,d\theta$ $\quad also\ r = R\,sec\,\theta$

Substituting these values in equation (1) we have

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{IR\,sec^2\,\theta\,d\theta\,cos\theta}{(R\,sec\,\theta)^2}$$

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{I\,cos\theta d\theta}{R}$$

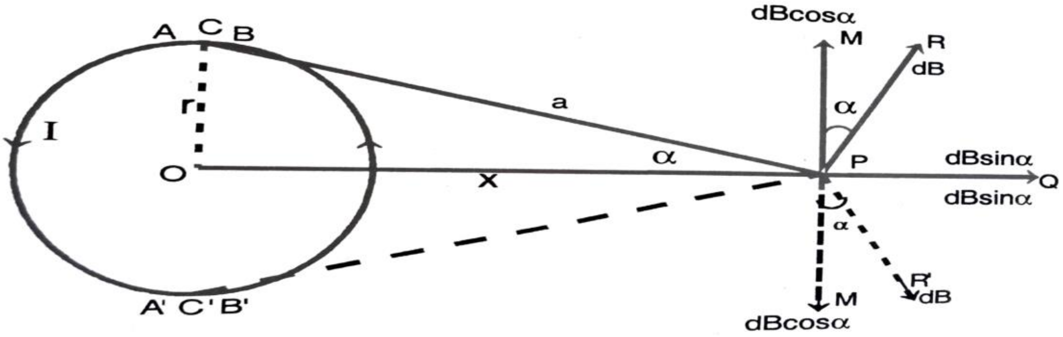
Hence the total magnetic field at the point P due to the entire conductor is given by

$$B = \int dB = \left(\frac{\mu_0}{4\pi}\right)\frac{I}{R}\int_{\theta_1}^{\theta_2} cos\theta d\theta = \left(\frac{\mu_0}{4\pi}\right)\frac{I}{R}\,(\sin\theta_2 + sin\theta_1)$$

For infinitely long conductor, $\theta_1 = \theta_2 = \frac{\pi}{2}$

$$\therefore \ \mathbf{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{\mathbf{I}}{\mathbf{R}}\,(\mathbf{1}+\mathbf{1}) \quad \textbf{or} \quad \mathbf{B} = \frac{\mu_0\,\mathbf{I}}{\mathbf{2\pi R}}$$

Expression for magnetic field at a point on the axis of a circular coil carrying current:



Consider a circular coil of radius r carrying a current Iand having 'n' number of turns. Let P be a point at a distance x from the centre O of the loop on the axis as shown in fig. AB and A′B′ are two diametrically opposite current elements each of length dl .The distance of the point P from these current elements be equal to 'a'

$$B= \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi I r^2}{a^3} \quad \text{or} \quad B = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi n I r^2}{(r^2+x^2)^{3/2}} \quad \ldots\ldots\ldots\ldots\ldots (1) \text{ along PX}$$

The direction of the magnetic field at any point on the axis of a circular loop carrying current is along the axis of the loop. If the plane of the coil is in the y-z plane, the magnetic field B is along the x-axis.

Discussion of special cases:

i) When x= 0, i.e. at the centre of the coil , the magnetic field is,

$$B= \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi n I r^2}{(r^2+0^2)^{3/2}} = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi n I r^2}{r^3} = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi n I}{r} = \frac{\mu_0 n I}{2r}$$

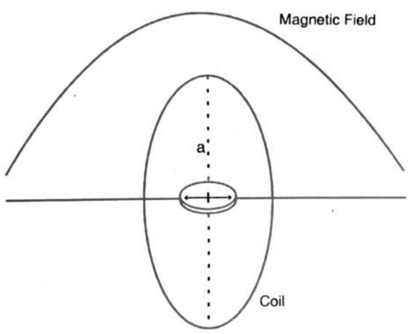
ii) If 'x' is too large compared to the value of r then, r may be neglected , now the eqn (1) becomes

$$B = \frac{\mu_0}{4\pi}\frac{2\pi\, n I r^2}{x^3} = \frac{\mu_0}{4\pi}\frac{2\pi r^2 n I}{x^3} = \frac{\mu_0}{4\pi}\frac{2\, A n I}{x^3} \qquad \text{since} \quad \pi r^2 = A$$

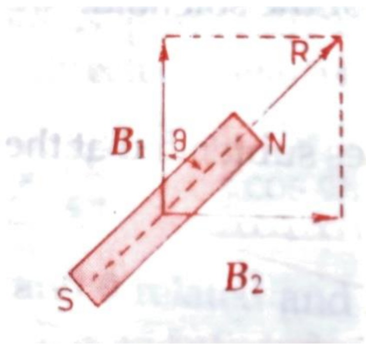
Now, nIA = m gives the **magnetic dipole moment** of the current carrying coil.

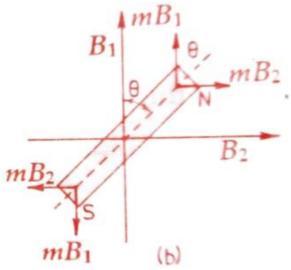
$$B = \frac{\mu_0}{2\pi}\frac{m}{x^3}$$

iii) From the equation (1), It is clear that at the center of the coil , The magnetic field at the centre of the loop is maximum and decreases along the axis on either side as the distance from the centre increases as shown in figure .



Tangent law: A magnet pivoted in a horizontal plane consisting of two uniform fields perpendicular to each other will set itself in the direction of the resultants of the fields. When there is only one uniform magnetic field, the magnet sets itself in the direction of B_1.





If another magnetic field B_2 is applied perpendicular to B_1, the magnet deflect through an angle θ and it sets itself in the direction of the resultant.(fig). The deflection θ is given by the relation

$$\mathbf{\tan\theta = \frac{B_2}{B_1} \quad \Rightarrow \; B_2 = B_1 \tan\theta} - - - - - - - - - - - - - - (1)$$

This is known as **tangent law.**

The equation (1) may be obtained more exactly in the following manner.

When the field B_2 is applied, the magnet is acted upon by two couples one due to B_1 being equal to $MB_1\sin\theta$ and other due to B_2 being equal to $MB_2 \,\mathrm{Sin}\,(90-\theta)$ i,e. $\boldsymbol{MB_2 \cos\theta}$ (fig2). These two couples are in opposite directions because couple due to B_2 tends to deflect it away from the original position and couple due to B_1 tends to restore it into its initial position. In the equilibrium position, the two couples balance each other,

$$\therefore \boldsymbol{MB_2 \cos\theta = MB_1 \sin\theta} \qquad \boldsymbol{ie \;\; B_2 = B_1 \tan\theta}$$

This principle is used in the construction of **Tangent Galvanometer.**

Note:

i) In practice $B_1 = B_H$, the horizontal; component of earth's magnetic field and B_2 = B is the magnetic field due to coil carrying current.

ii) If B_2 = B is called **the deflecting field** then B_H is the controlling field.

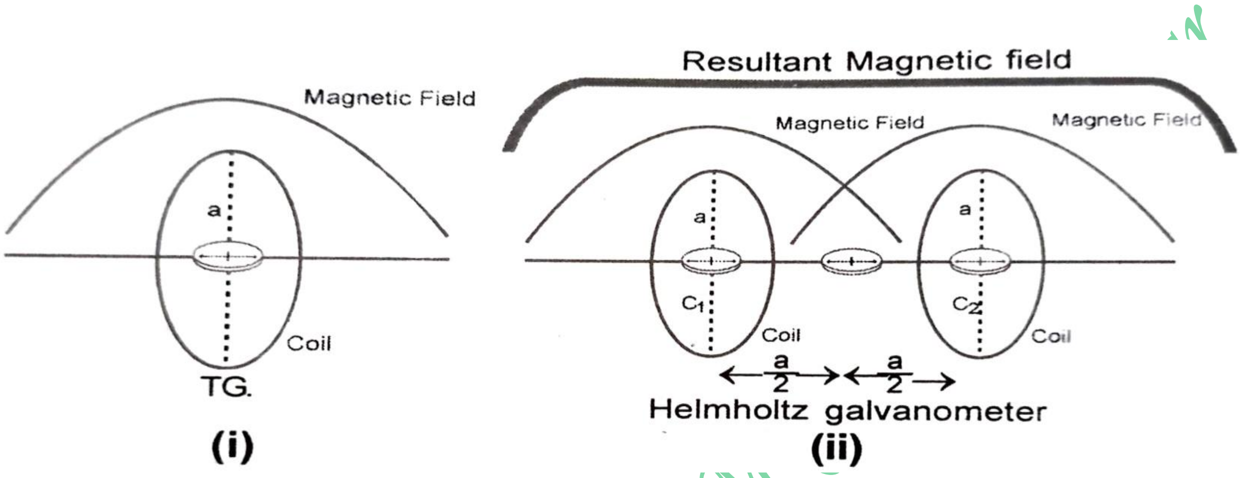
iii) The equation $\boldsymbol{B = B_H \tan\theta}$ holds well if and only if both B and B_H are uniform in the region in which the magnet is suspended.

This is the reason why a short magnetic needle is used in the magnetic compass box.

Helmholtz Galvanometer:

In a tangent galvanometer, the magnetic field produced due to the current in a coil is uniform over a very small region at the centre of the coil. This requires very short magnetic needle to be pivoted at the centre of coil so that it can rotate in small region of uniform field. Helmholtz showed that this difficulty could be overcome and the uniform field over a considerable region could be produced by using two coils instead of one as in the case of a tangent galvanometer.

Helmholtz galvanometer is the modified form of TG.



Construction:

It consists of two coils of radius 'r' each having an equal number of turns n. The coils are situated co-axially with their plane faces parallel to each other. The distance between their centers is equal to the radius 'r' of either coil. The coils are connected in series so that the direction of the current in both the coils is the same and hence the resultant field at the centre space of them is twice due to each coil. The magnetic needle is suspended at the central point of the two coils.

Principle:

When the current is passed through two coils, two magnetic fields are produced as shown in the fig. The coils are so connected that the direction of the flow of current in both the coils is the same and hence the intensity of magnetic field due to these coils is also in the same direction at any point on their common axis C_1 C_2. As we move from C_1 towards C_2 the magnetic field B increases due to C_2 coils and that due to C_1 decreases simultaneously. As a result, there is a uniform resultant magnetic field over the region between the coils.

The condition to get an uniform field over the region is that the rate of increase of field due to one coil must be equal to the rate of decrease of the field due to other coil in the same direction. If this condition is to be satisfied, then the rate of change of resultant field B with respect to the distance from one of coils should remain constant.

$$\text{i.e} \quad \frac{dB}{dx} = a\ constant$$

$$\text{i.e} \qquad \frac{d^2 B}{dx^2} = 0$$

We know that the magnetic field produced at any point on the axis of a circular coil is given by

$$B = \frac{\mu_0}{4\pi} \frac{2\pi nlr^2}{(r^2 + x^2)^{3/2}} \quad \text{tesla}$$

$$= \frac{\mu_0}{2} \frac{nlr^2}{(r^2 + x^2)^{3/2}}$$

$$= \frac{\mu_0 nla^2}{2} (r^2 + x^2)^{-3/2}$$

Differentiating this we have

$$\frac{dB}{dx} = \frac{\mu_0 nlr^2}{2} [\frac{-3}{2} (r^2 + x^2)^{-5/2} 2x]$$

$$\frac{d^2 B}{dx^2} = -\frac{3\mu_0 nlr^2}{2} [(a^2 + x^2)^{-5/2} \frac{-5}{2} x(a^2 + x^2)^{-7/2} 2x]$$

$$5x^2 (r^2 + x^2)^{-7/2} = (r^2 + x^2)^{-5/2}$$

$$5x^2 = r^2 + x^2$$

$$4x^2 = r^2$$

$$x^2 = \frac{r^2}{4}$$

$$x = \pm \frac{r}{2}$$

Thus two coils must be kept at a distance of radius of the coil and the magnetic needle must be suspended at a distance of half the radius from each coil.

Since Helmholtz galvanometer consists of two coils, the resultant field at the point where the needle is suspended is given by (2B)

$$\therefore B = 2 \left[\frac{\mu_0 nlr^2}{2(r^2 + x^2)^{3/2}}\right]$$

$$B = \frac{\mu_0 nlr^2}{[r^2 + \frac{r^2}{4}]^{3/2}} \qquad \because x = \frac{r}{2}$$

$$\mathbf{B} = \frac{8\mu_0 nl}{5\sqrt{5}r} \ldots \ldots \ldots \ldots \ldots (1)$$

This field acts along the common axis of the coils.

Working:

Before passing the current the planes of the coils are set into magnetic meridian. The distance between the coils must be equal to the radius of the coil. The deflection magnetometer must be kept between the coils so that its needle is at the distance of a/2 from each coil. When the current is passed through the coil, the needle gets deflected through the angle θ.

According to the tangent law we have

Now B = B_H tanθ -----------(2)

Eqn. (1) = Eqn. (2)

$$\frac{8\mu_0 nI}{5\sqrt{5}r} = B_H \tan\theta$$

$$\therefore I = \frac{5\sqrt{5}rB_H}{8\mu_0\, n}\tan\theta$$

$$I = K\tan\theta$$

Where $K = \frac{5\sqrt{5}rB_H}{8\mu_0\, n}$ is known as the **reduction factor of the galvanometer.**

AMPER'S CIRCUITAL LAW:

Statement: It states that **the line integral of magnetic field B around any closed curve in air or vacuum is equal to μ_0 times the net current I through the area bounded by the curve.**

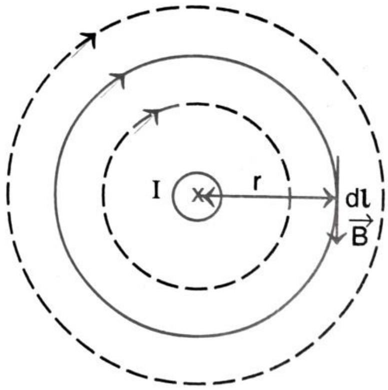
i.e $\oint \boldsymbol{B}.\boldsymbol{dl} = \mu_0 \mathbf{I}$. Where B is the magnetic field due to the current I.

Note: This law plays the same in magnetostatics as Gauss' law does in electrostatics.

Proof: Consider a long straight conductor carrying current I perpendicular to the plane of the paper so that the current flows inwards

The magnetic field at a distance r is given by

$\mathbf{B} = \frac{\mu_0\, \mathbf{I}}{2\pi \mathbf{r}}$ and its direction is tangent to the circle of radius r.



The field is constant at every point on the circle and parallel to the current element dl.

The line integral is given by,

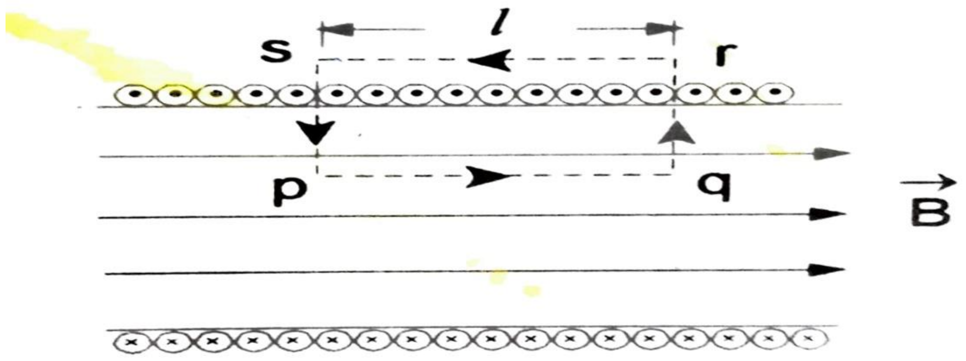
$$\oint \boldsymbol{B}.\boldsymbol{dl} = \oint \frac{\mu_0\, \mathbf{I}}{2\pi \mathbf{r}}.\boldsymbol{dl} = \frac{\mu_0\, \mathbf{I}}{2\pi \mathbf{r}}\oint \boldsymbol{dl} = \frac{\mu_0\, \mathbf{I}}{2\pi \mathbf{r}}(2\pi \mathbf{r}) = \mu_0 \mathbf{I}$$

$\therefore \oint \boldsymbol{B}.\boldsymbol{dl} = \mu_0 \mathbf{I}$ which is independent of radius r. Ampere's circuital law is true for any assembly of current and any closed curve.

APPLICATIONS OF AMPERE CIRCUITAL LAW:

1. MAGNETIC FIELD DUE TO A SOLENOID CARRYING CURRENT:

A solenoid consists of a long insulated wire wound in the form of a helix where neighboring turns are closely spaced. Each turn of wire in the solenoid can be regarded as circular loop. When current flows through the solenoid, the net magnetic field is the vector sum of the fields due to all the turns. A long solenoid means that the length of the solenoid is large compared to its radius.



Let n be the number of turns per unit length of a long solenoid and I be the current flowing through the solenoid. Consider a rectangular Amperian loop pqrs near the middle of the solenoid as shown in fig. The line integral of magneticf field B along the path pqrs is

$\oint_{pqrs} \vec{B} \bullet \vec{dl} = \int_{pq} \vec{B} \bullet \vec{dl} + \int_{qr} \vec{B} \bullet \vec{dl} + \int_{rs} \vec{B} \bullet \vec{dl} + \int_{sp} \vec{B} \bullet \vec{dl}$ -------------(1)

Let pq = l. For path pq B and dl are along the same direction.

$\therefore \int_{pq} \vec{B} \bullet \vec{dl} = \int B.dl = Bl$

For the path qr and sp B and dl are mutually perpendicular

$$\therefore \int_{qr} \vec{B} \bullet \vec{dl} = \int_{sp} \vec{B} \bullet \vec{dl} = \int B.dl\, cos90 = 0$$

For path rs, B = 0 (since field is zero outside the solenoid)

$$\therefore \int_{rs} \vec{B} \bullet \vec{dl} = \mathbf{0}$$

Equation 1 becomes

$\oint_{pqrs} \vec{B} \bullet \vec{dl} = \int_{pq} \vec{B} \bullet \vec{dl} = \int B.dl = Bl$ -----------(2)

From Ampere's circuital law,

$\oint_{abcd} \vec{B} \bullet \vec{dl} = \mu_o$ x net current enclosed by the path

i .e. $Bl = \mu_o$ x nlI

$\therefore B = \mu_o nI$ ------------------ (3)

Note: The magnetic field at a point near the end of the solenoid is found to be equal to = $\frac{\mu_o nI}{2}$

2. MAGNETIC FIELD DUE TO CURRENT IN A TOROID:

The toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. In fact, a toroid is an endless solenoid in the form of a ring or a solenoid bent into a circular shape to close on itself.

Let n be the number of turns per unit length of toroid and I be current flowing through it. Point P is within the toroid while Q is inside and point R outside. By symmetry, direction of B at any point is tangential to a circle drawn through that point with same centre as that of toroid. The magnitude of B at any point of such a circle will be constant.

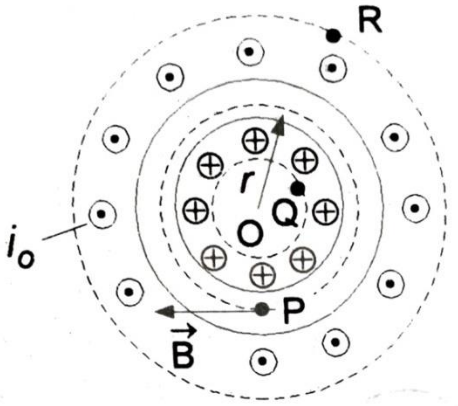
Let us consider a point p within the toroid, let us draw a circle of radius r through it.

Appling Ampere's circuital law to this circle we have

$\oint \boldsymbol{B.dl} = \mu_o \mathbf{I}$ where I is the net current enclosed by the circle.

Now $\oint \boldsymbol{B.dl} = \mathbf{B(2\pi r)}$ and $I = n\,I_0$

Where '**n**' is the total no of turns in the toroid and $\mathbf{I_0}$ is the current in each turn of the toroid.



Therefore, equation 1 becomes,

$\mathbf{B(2\pi r)} = \mu_o\, n\, I_0$

Or, $\mathbf{B} = \frac{\mu_0}{2\pi}\frac{nI_0}{r}$

Thus the magnetic field B varies with r.

If '*l*' is the mean circumference of the toroid then, $l = 2\pi r$ so that

$$\mathbf{B} = \frac{\mu_0 n I_0}{l}$$

The field B at an inside point Q is zero because there is no current enclosed by the circle through Q. The field B at an outside point such as R is also zero because net current enclosed in the circle through r will be zero. This is because each term of the winding passes twice through this area enclosed by the circle, carrying equal currents in opposite directions. Thus, the field of a toroid is zero at all point except within the core.