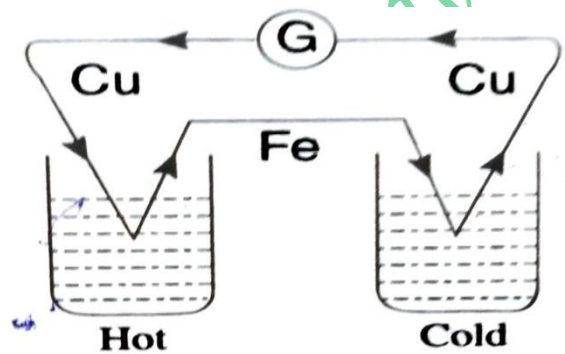
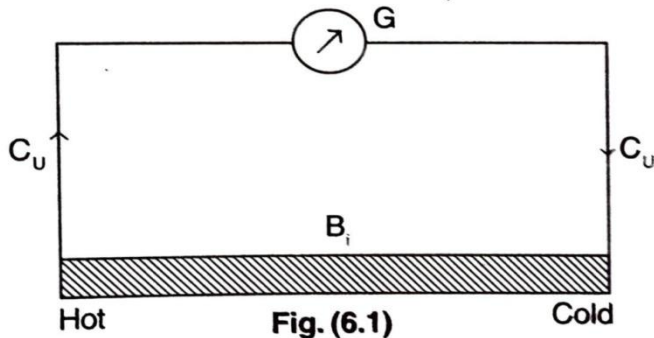


## THERMOELECTRICITY

### SEEBACK EFFECT

In 1821 Seeback found that a current flows in a circuit consisting of two dissimilar metals when one junction is heated while the other junction kept cold. This was remarkable experiment because *no cell* was used in the circuit. He connected a plate of bismuth between copper wires connected to a galvanometer.



He found that if one of the junctions heated while the other was kept cold, then a current flowed through the galvanometer. He repeated his experiment by taking a thermocouple of *Fe* and *Cu*.

If both the junctions are at  $0^{\circ}\text{C}$ , there is no deflection in the galvanometer. When one of the junctions is kept constant at  $0^{\circ}\text{C}$ , i.e., at the temperature of the melting point of ice and the other junction is heated gradually, current flows in the circuit. It was found that current flows from copper to iron at the hot junction and iron to copper at the cold junction. The current increases until the hot junction is at a temperature  $270^{\circ}\text{C}$ . If the heating is continued beyond the  $270^{\circ}\text{C}$ , the current decreases and finally the current is zero at  $540^{\circ}\text{C}$ .

It was discovered by *Cumming* in 1823 that on increasing the temperature of hot junction beyond  $540^{\circ}\text{C}$ , the direction of the current is reversed. It flows from iron to copper through the hot junction and copper to iron through the cold junction.

The current produced in this way without the use of a cell or a battery is known as **thermo-electric current** and The EMF produced in this way is called **thermo-EMF**, this branch of electricity is known as **thermo-electricity**. The effect is known as **Seeback effect**.

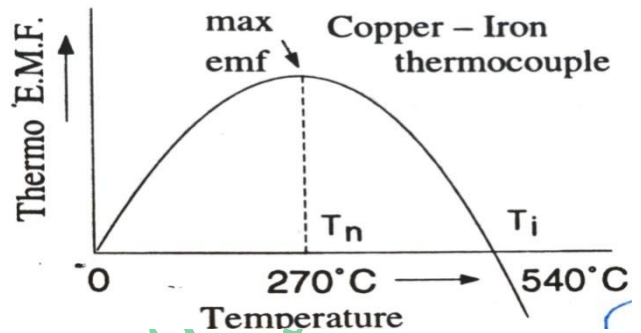
### EXPLANATION OF SEEBACK EFFECT

The free electrons inside a material can be considered to constitute an electron gas. The electron density is different in different metals. When two such dissimilar metals are joined together, electrons move from the metal of higher electron density to the metal of lower density. Thus potential difference is set up at the junction which is known as **contact potential difference**. When the two junctions are at the same temperature there is no potential difference between the junctions, so there is no flow of current in the

circuit. If one of the junctions is at higher temperature, emf is set up in the circuit which results in the flow of current.

The temperature of the junction at which maximum current flows in a circuit is known as the **neutral temperature** for that couple. The neutral temperature for a given thermocouple is fixed and remains constant whatever may be the temperature of the cold junction.

If a graph is plotted between the temperature of the hot junction and the thermo-EMF, the cold junction being kept at  $0^\circ\text{C}$ , the graph is a parabolic curve. The thermo-EMF  $E$  varies with temperature according to  $E=at+bt^2$ , where  $a$  and  $b$  are constants. The point  $A$  represents the neutral temperature. The point  $B$  is the **Temperature of inversion** beyond which the direction of current is reversed. The temperature of inversion is not fixed. It is as much above the neutral temperature as cold junction is below the neutral temperature.



Suppose the temperature of the cold junction= $T_c$  Neutral temperature= $T_n$  Temperature of inversion= $T_i$

$$\text{Then, } T_n - T_c = T_i - T_n, \quad T_n = \left(\frac{T_c + T_i}{2}\right)$$

In the case of Cu-Fe thermocouple, if the cold junction is at  $100^\circ\text{C}$ , the temperature of inversion= $440^\circ\text{C}$ .

The following is the list of metals, out of which, if any two of them form a thermocouple, the current flows across the cold junction from the metal occurring first in the series to the one occurring later.

**Selenium, antimony, iron, cadmium, zinc, silver, gold, lead, mercury, copper, platinum, nickel, bismuth.**

The metals preceding lead are taken to be thermo-electrically negative and succeeding lead are thermo-electrically positive. The farther the metals are apart in the list, the greater will be the thermo-EMF produced in the circuit for a given difference of temperature between the two junctions. As antimony and bismuth are quite distant, the thermo-EMF produced is large. Due to this reason antimony-bismuth thermocouple is preferred. Current flows from antimony to bismuth across the cold junction ( $A \rightarrow B$  at  $C$ ).

### LAW OF SUCCESSIVE TEMPERATURES

**Law of Successive Temperatures states that for a given thermocouple, the thermo-EMF for any number of successive temperatures is the sum of the thermo-EMF for number of successive steps into which the given range of temperature may be divided.**

Suppose,  $T_1, T_2, T_3, T_4, \dots, T_n$  are successive temperatures between  $T_1$  and  $T_n$ ,

$$\text{Then, } E_{T_1}^{T_n} = E_{T_1}^{T_2} + E_{T_2}^{T_3} + E_{T_3}^{T_4} \dots \dots \dots E_{T_{n-1}}^{T_n}$$

Suppose for a given thermocouple, the thermo-EMF between  $20^\circ\text{C}$  and  $100^\circ\text{C}$  is  $E_1$ , between  $20^\circ\text{C}$  and  $50^\circ\text{C}$  is  $E_2$  and between  $50^\circ\text{C}$  and  $100^\circ\text{C}$  is  $E_3$ . It can be proved experimentally,  $E_1 = E_2 + E_3$ .

## LAW OF INTERMEDIATE METALS OR SUCCESSIVE CONTACTS

Law of intermediate metals states that, insertion of another metal into a thermocouple circuit does not change the total thermo-EMF provided that the added metal is entirely at the temperature of the part of the circuit where it is inserted.

In general, the law states that, if at a given temperature, a number of metals are in successive contact so as to form a chain of elements connected in series, the thermo-EMF between the extreme elements, if placed in direct contact, is the sum of the thermo-EMF between successive adjacent elements.

Suppose the metals A, B and C are in contact,  $E_A^C = E_A^B + E_B^C$

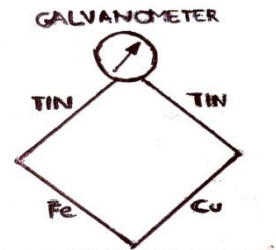
where  $E_A^B, E_B^C, E_A^C$  denote the EMFs, for circuits with metals A and B, B and C and A and C respectively for given fixed junction temperature.

Bequerel verified the truth of the law. He joined Fe, Cu and Tin to a galvanometer as shown in figure. He observed the deflection in the galvanometer when one junction was heated at  $20^\circ\text{C}$  while the other two were kept at  $0^\circ\text{C}$ . The deflections observed were.

27.96 When Fe-Cu junction was heated,  $E_{\text{Fe}}^{\text{Cu}} = 27.96$

3.50 When Cu-Tin junction was heated,  $E_{\text{Cu}}^{\text{Tin}} = 3.50$

and 31.24 When Fe-Tin junction was heated,  $E_{\text{Fe}}^{\text{Tin}} = 31.24$



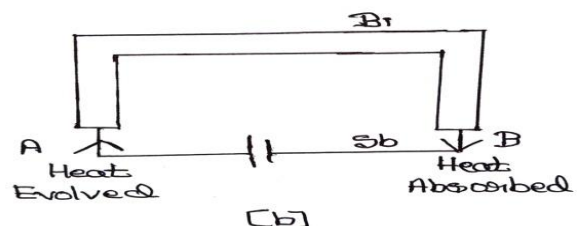
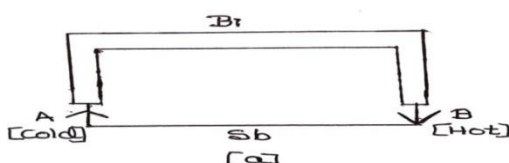
$E_{\text{Fe}}^{\text{Cu}} + E_{\text{Cu}}^{\text{Tin}} = 27.96 + 3.50 = 31.46$  Which is approximately the same as for  $E_{\text{Fe}}^{\text{Tin}}$

## THERMOELECTRIC POWER $\left(\frac{dE}{dT}\right)$

If the temperature of the hot junction is varied keeping the cold junction at a constant temperature, the thermo emf of the couple also changes. **The rate of change of thermo emf with temperature of the hot junction is called thermo electric power of the couple at the particular temperature.** It is given by  $\frac{dE}{dT}$ . The slope of tangent to the curve E against T gives the **thermo electric power**.

## PELTIER EFFECT:

In 1834, peltier a French physicist discovered that when an electric current is passed through a thermocouple heat is evolved at one junction and absorbed at the other junction. Thereby one junction gets heated and the other gets cooled. It is the reverse process of Seeback effect.



**The process of absorption or evolution of heat energy at a junction between two dissimilar metals when a current flows through the junction is called the Peltier effect.**

In an *Sb-Bi* couple when the junction *B* is heated, and *A* is kept cold, current flows from *Sb* to *Bi* at the junction *A* and from *Bi* to *Sb* at the junction *B* [fig. (b)]. When a battery is placed in the circuit and both the junction *A* and *B* are kept at the same temperature, heat is evolved at the junction *A* and absorbed at the junction *B* provided the current flows from *Sb* to *Bi* at the junction *A* and from *Bi* to *Sb* at the junction *B* [fig(a)]. When the direction of the current is reversed, the heat is evolved at the junction *B* and absorbed at *A*. this shows that peltier effect is reversible. It is not to be confused with joule effect which is irreversible.

#### **EXPLANATION OF PELTIER EFFECT:**

When two metals are in contact the potential difference is set up between them. If the battery is connected in the circuit, an external potential difference is also set up at the two ends of the junction. Then work has to be done to move a unit charge along the direction of the electric field opposite to it. If the charge moves to overcome the rise of potential heat is absorbed and hence there is a fall of temperature. On the other hand if the charge moves along the fall of potential, then heat is developed and there is a rise of temperature.

#### **DISTINCTION BETWEEN PELTIER EFFECT AND JOULE EFFECT:**

<b>Peltier effect</b>	<b>Joule effect</b>
1. The effect is reversible.	1. It is irreversible.
2. Heat is evolved at one end junction and absorbed at the other end.	2. Heat is always evolved.
3. It takes place only at junction.	3. It takes place throughout the conductor.
4. Heat evolved or absorbed is proportional to the current. i.e. $H = \pi It$ .	4. Heat evolved is proportional to the square of the current. i.e. $H = I^2 Rt$ .

#### **Peltier coefficient:**

The peltier coefficient of a junction is defined as the amount of heat energy absorbed or evolved at a junction when a unit current flows for one second. If the peltier coefficient is represented as  $\pi$  and current of  $I$  ampere is passed for  $t$  seconds,

Then energy evolved or absorbed =  $\pi It$  Joule.

If  $E$  is the emf of the junction then,

$$EIt = \pi It \quad \text{or} \quad E = \pi$$

Peltier coefficient is not constant but it depends on temperature of the junction. In the case of thermocouple

If  $\pi_2$  and  $\pi_1$  are the peltier coefficient at the junction where energy is absorbed and evolved respectively, then thermo-EMF,  $E = (\pi_2 - \pi_1)$

**Determination of the Peltier coefficient:**

A junction of a thermocouple is dipped in water and a current of  $I$  ampere is passed for known time  $t$ , if the amount of heat produced =  $H_1$

$$\text{Then, } H_1 = I^2 R t + \pi I t \quad \dots \dots \dots (1)$$

Where  $I^2 R t$  is the heat produced due to Joules effect and  $\pi I t$  is due to Peltier effect.

On reversing the current through the circuit, suppose the heat produced =  $H_2$ .

$$\text{Then } H_2 = I^2 R t - \pi I t \quad \dots \dots \dots (2)$$

Because Peltier effect is reversible

Subtracting (2) from (1)

$$H_1 - H_2 = 2 \pi I t \quad \text{or} \quad \pi = \frac{(H_1 - H_2)}{2 I t} \quad \dots \dots \dots (3)$$

Hence peltier coefficient can be calculated.

**Thermodynamics of Peltier effect (Discovery of Thomson's effect)**

Consider a thermocouple of any two metals  $A$  and  $B$  (fig). Its cold junction is at temperature  $T_1$  and hot junction is at temperature  $T_2$  and thermoelectric current is produced. It means, when thermoelectric current is passed round the junction, it absorbs heat at temperature  $T_2$  the hot junction and evolves heat at temperature  $T_1$  at the cold junction. The heat absorbed at hot junction is greater than heat evolved at cold junction. It means that in a thermocouple when heat is absorbed at hot junction a part of it is converted into electrical energy and remaining amount of heat is rejected at cold junction. Similarly if the current is passed through the thermocouple heat is absorbed at one junction and evolved at the other junction. These two effects reversible.

Thus a thermocouple acts like a reversible heat engine which takes heat from the source at the hot junction, does some work in during the current in the circuit and rejects the remaining heat to the sink, i.e. at cold junction.

Suppose one ampere of current flows for one second and peltier coefficients for the given thermocouples are  $\pi_2$  at temperature  $T_2$  and  $\pi_1$  at temperature  $T_1$ .

Heat energy absorbed at temperature  $T_2$  at the hot junction.

$$= \pi_2 I t = \pi_2 \times 1 \times 1 = \pi_2 \quad \text{joule}$$

Heat energy evolved at temperature  $T_1$  at the cold junction.

$$= \pi_1 I t = \pi_1 \times 1 \times 1 = \pi_1 \quad \text{joule}$$

$$\therefore \text{Net energy absorbed} = (\pi_2 - \pi_1).$$

This energy is used in setting up a thermo emf in the circuit =  $E$  volt

$$\therefore E = (\pi_2 - \pi_1)$$

The Peltier effect is reversible just like Carnot's reversible engine.

According to Carnot's engine,

$$\frac{H_1}{T_1} = \frac{H_2}{T_2} \quad \text{here } H_1 = \pi_1 \quad \text{and } H_2 = \pi_2$$

$$\frac{\pi_1}{T_1} = \frac{\pi_2}{T_2} \Rightarrow \frac{\pi_2}{\pi_1} = \frac{T_2}{T_1}$$

Add -1 on both side,

$$\frac{\pi_2}{\pi_1} - 1 = \frac{T_2}{T_1} - 1$$

$$\frac{\pi_2 - \pi_1}{\pi_1} = \frac{T_2 - T_1}{T_1}$$

$$\pi_2 - \pi_1 = \frac{\pi_1}{T_1} (T_2 - T_1)$$

**But,**  $\pi_2 - \pi_1 = E$

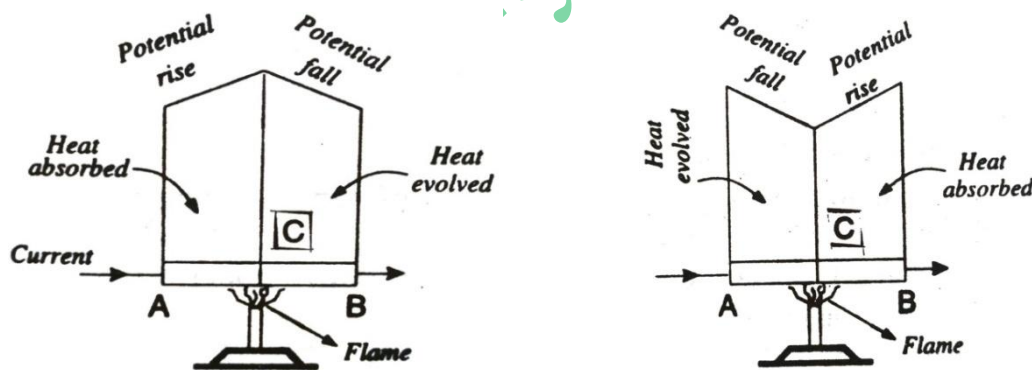
$$\therefore E = \frac{\pi_1}{T_1} (T_2 - T_1)$$

$$\therefore E \propto (T_2 - T_1)$$

Thus the thermo emf is directly proportional to the difference of temperature  $T$  between hot and cold junction. If a graph is plotted between  $E$  and  $T$  it should be a straight line.

But it has been found experimentally that the graph between  $E$  and  $T$  is not a straight line but it is a parabolic curve and  $E$  is not  $\propto (T_1 - T_2)$ . This shows that PE alone cannot explain the thermoelectric phenomena in a thermocouple. This led to the discovery of **Thomson effect**.

**THOMSON'S EFFECT:** According to Thomson when a current flows through an unequally heated conductor, there is an evolution or absorption of heat not only at the junctions but evolution or absorption also takes place throughout the conductor. This effect is known as **Thomson's effect**.



#### POSITIVE AND NEGATIVE THOMSON'S EFFECT:

Consider a thick copper bar AB heated at its centre. If no current flows through the conductor, the points A & B equidistant from the center are at same temperature. Suppose the current is sent through the bar in the direction along AB, it is found that the point A is at lower temperature and B is at higher temperature. That is heat is absorbed from A to the center and heat is evolved from center to B. There is a transfer of heat in the direction of current. This effect is known as the **positive Thomson Effect**. Similar effect is also observed in Cd, Zn, Ag and Sb.

In case of iron bar, if current is flows in the direction from A and B Fig (2) It is found that the point A is at higher temperature and B is at lower temperature. The transference of heat takes place in the direction opposite to the direction of current. This effect is known as the **negative Thomson Effect**. Similar effect is also observed in Pt, Bi, Co, Ni and Hg.

In case of lead, Thomson effect is zero. That is why lead is taken as one of the metal to form a thermocouple.

#### **EXPLANATION OF THOMSON EFFECT:**

It can be explained on the basis of free electron theory of metals, when metallic bar is unequally heated; temperature gradient is set up in that single bar. Naturally, the electrons in the hotter parts of the conductor are more energetic than those in cooler parts. Hence the electrons move from hotter to colder parts of the conductor producing higher potential at the hotter portions and lower potential at the colder portions. If the electric charge is sent through the conductor, work is to be done along the direction of potential difference or opposite to it. As a result heat energy is either absorbed or evolved. Due to the difference of these two energies the Thomson emf is produced.

#### **THOMSON'S CO-EFFICIENT OF A METAL:**

Thomson coefficient of a metal is defined as the **amount of heat energy absorbed or evolved when a unit quantity of electric charge flows in the metal between two points having temperature difference of 1°C**, it is denoted by  $\sigma$ .

It is also defined as the potential difference set up between two points for unit temperature difference.

Consider Q coulomb of charge is flowing in a metal between two points having temperature difference of 1°C, and then we have.

Heat energy absorbed or evolved =  $\sigma Q$  joule

If E volt is the Thomson is emf, set up between these points, then this energy must be equal to EQ joules

$$\therefore \sigma Q = EQ \quad \text{or} \quad \sigma = E$$

**S.I unit of Thomson coefficient is  $\text{JC}^{-1}\text{K}^{-1}$ .**

Thomson Coefficient can also define as the **heat energy evolved or absorbed when one ampere current flows for one second between two points of the conductor at a temperature difference of one degree.**

If two points are at temperature  $T_1$  and  $T_2$  the energy absorbed or evolved when on ampere current flows for t second. It is given by

$$= \left( \int_{T_1}^{T_2} \sigma dT \right) I t \quad \text{If } I = 1\text{A}, t = 1\text{s then, Thomson emf} = \sigma dT$$

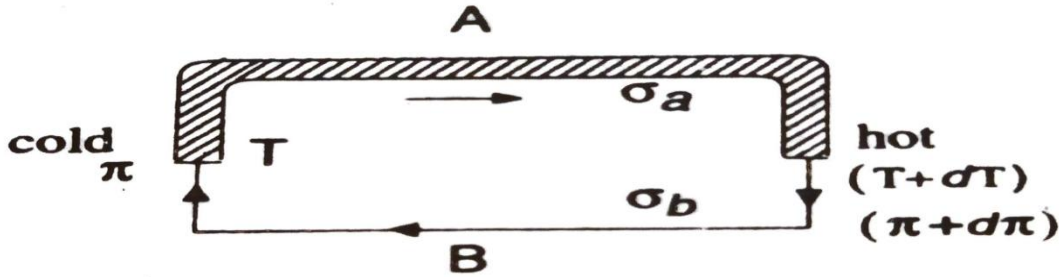
**Thermoelectric power**  $\left( \frac{dE}{dT} \right)$

If the temperature of the hot junction is varied keeping the cold junction at a constant temperature, the thermo emf of the couple also changes. The rate of change of thermo emf with temperature of the hot junction is called **Thermo electric power** of the couple at the particular temperature. It is given by  $dE/dT$ .

The slope of tangent to the curve E against T gives the **thermo electric power**.

### Thermodynamics of Thermocouple (Expressions for Peltier and Thomson coefficients).

Consider a thermocouple consisting of two metals A and B. Let  $T$  and  $T+dT$  be the temperatures of the cold and hot junction respectively (figure). Let  $\pi$  and  $\pi+d\pi$  be the Peltier coefficients for the pair at the cold and hot junctions. Let  $\sigma_a$  and  $\sigma_b$  be the Thomson coefficients for the metals A and B respectively, both taken as positive. When a charge flows through the thermocouple, heat will be absorbed and evolved at the junctions due to Peltier effect and all along the metal due to Thomson effect.



Let 1 coulomb of charge flow through the thermocouple in the direction from A to B at the hot junction.

Heat energy absorbed due to Peltier effect at the hot junction =  $(\pi+d\pi)$  joule

Heat energy evolved due to Peltier effect at the cold junction =  $\pi$  joule

Heat energy absorbed in the metal A due to Thomson effect

$$= \sigma_a dT \quad \text{joule}$$

Heat energy evolved in the metal B due to Thomson effect

$$= \sigma_b dT \quad \text{joule}$$

$\therefore$  Net heat energy absorbed in the thermocouple

$$= (\pi + d\pi - \pi) + (\sigma_a dT - \sigma_b dT)$$

$$= d\pi (\sigma_a - \sigma_b) dT$$

This energy is used in establishing a P.D.  $dE$  in the thermocouple

$$\therefore dE = d\pi + (\sigma_a - \sigma_b) dT \quad \dots\dots\dots (1)$$

Since the Peltier Thomson effects are reversible, the thermocouple acts as a reversible heat engine. Here,

(i) The heat energy  $(\pi + d\pi)$  joules are absorbed from the source at  $(T + dT)$  K and  $\sigma_a dT$  joule is absorbed in metal A at mean temperature  $T$  K.

(ii) Also  $\pi$  joule is rejected to sink at  $T$  K and  $\sigma_b dT$  joule is given out in metal B at the mean temperature  $T$  K.

Applying Carnot's theorem, we have

$$\frac{\pi + d\pi}{T + dT} + \frac{\sigma_a dT}{T} = \frac{\pi}{T} + \frac{\sigma_b dT}{T}$$

Or 
$$\frac{\pi + d\pi}{T + dT} - \frac{\pi}{T} = \frac{(\sigma_b - \sigma_a) dT}{T}$$

Or 
$$\frac{\pi T + d\pi T - \pi T - \pi dT}{T(T+dT)} = \frac{(\sigma_b - \sigma_a) dT}{T}$$

Or 
$$d\pi \cdot T - \pi \cdot dT = (\sigma_b - \sigma_a) dT (T + dT)$$



$$\text{Or } d\pi \cdot T - \pi dT = (\sigma_b - \sigma_a)TdT + (\sigma_b - \sigma_a)dT^2$$

$$\text{Or } (d\pi \cdot T - \pi dT) = (\sigma_b - \sigma_a)T \cdot dT$$

[neglecting  $(\sigma_b - \sigma_a)dT^2$ ]

$$\text{Or } T[d\pi + (\sigma_a - \sigma_b)dT] = \pi dT$$

$$\text{But } d\pi + (\sigma_a - \sigma_b)dT = dE \quad \text{from Eq. (1)}$$

$$\therefore TdE = \pi dT$$

$$\text{Or } \pi = T \frac{dE}{dT} \dots \dots \dots (2)$$

The quantity  $\frac{dE}{dT}$  is called the thermoelectric power (P)

Thermoelectric power (P) is defined as the thermo emf per unit difference of temperature between the junctions.

$\therefore$  Peltier coefficient = Absolute temperature  $\times$  thermoelectric power

$$\text{Differentiating Eq. (2), } \frac{d\pi}{dT} = T \frac{d^2E}{dT^2} + \frac{dE}{dT}$$

Substituting the value of  $(dE/dT)$  from Eq. (1),

$$\frac{d\pi}{dT} = T \frac{d^2E}{dT^2} + \frac{d\pi}{dT} + (\sigma_a - \sigma_b)$$

$$\text{Or } (\sigma_a - \sigma_b) = -T \cdot \frac{d^2E}{dT^2}$$

$$\text{Or } (\sigma_b - \sigma_a) = T \cdot \frac{d^2E}{dT^2} \dots \dots \dots (3)$$

if the first metal in the thermocouple is lead, then  $\sigma_a = 0$

$$\therefore \sigma_b = T \cdot \frac{d^2E}{dT^2} \dots \dots \dots (4)$$

Thomson coefficient = absolute temperature of cold junction  $\times$  First derivative of thermoelectric power.

$$\text{From Eq. (3), } \frac{d^2E}{dT^2} = \frac{(\sigma_b - \sigma_a)}{T} \quad \text{or} \quad \frac{d}{dT} \left( \frac{dE}{dT} \right) = \frac{(\sigma_b - \sigma_a)}{T}$$

Putting  $dE/dT$  from Eq. (2), we have

$$\frac{d}{dT} \left( \frac{\pi}{T} \right) = \frac{(\sigma_b - \sigma_a)}{T}$$

$$\text{Or } \frac{d}{dT} \left( \frac{\pi}{T} \right) - \left( \frac{\sigma_b - \sigma_a}{T} \right) = 0 \dots \dots \dots (5)$$

This gives the relation between Peltier and Thomson's coefficients.

**THERMO-ELECTRIC DIAGRAM :** A thermocouple is formed from two metals A and B. The difference of temperature of the junction is  $TK$ . The thermo emf is given by the equation  $E = aT + bT^2$

A graph between  $E$  and  $T$  is a parabola.  $\frac{dE}{dT} = a + 2bT$

$\frac{dE}{dT}$  is called thermoelectric power.

A graph between thermoelectric power ( $\frac{dE}{dT}$ ) and difference of temperature  $T$  is a straight line. This graph is known as thermoelectric power line or the thermo-electric diagram or Tait diagram. Thomson coefficient of lead is zero. So generally thermo-electric lines are drawn with lead as one metal of the thermo-couple. The thermoelectric line of a  $Cu-Pb$  couple has a positive slope while that of  $Fe-Pb$  couple has a negative slope. Figure (1) shows the power lines for a number of metals.

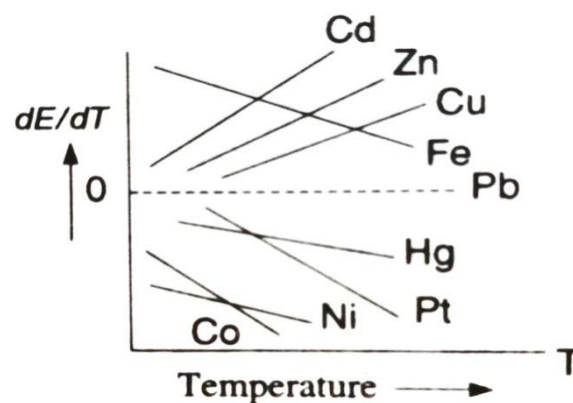


Figure (1)

### Uses of Thermoelectric Diagram

#### (i) Determination of total emf:

$MN$  Represents the thermoelectric power line of a metal like copper coupled with lead figure (2)  $MN$  has a positive slope.

Let  $A$  and  $B$  be two points corresponding to temperature  $T_1$  K and  $T_2$  K respectively along the temperature-axis. Consider a small strip  $abcd$  of thickness  $dT$  with junctions maintained at temperature  $T$  and  $(T+dT)$ .

The emf developed when the two junctions of the thermocouple differ by  $dT$  is

$$dE = dT \cdot \left(\frac{dE}{dT}\right) = \text{Area } abdc$$

Total emf developed when the junction of the couple are at temperature  $T_1$  and  $T_2$  is

$$E_s = \int_{T_2}^{T_1} dT \left(\frac{dE}{dT}\right) = \text{Area } ABDC$$

#### (ii) Determination of peltier emf:

Let  $\pi_1$  and  $\pi_2$  be the peltier coefficient for the junction of the couple at the temperature  $T_1$  and  $T_2$  respectively.

The peltier coefficient at hot junction ( $T_2$ ) is

$$\pi_2 = T_2 \left(\frac{dE}{dT}\right)_{T_2} = OB \times BD = \text{Area } OBDF$$

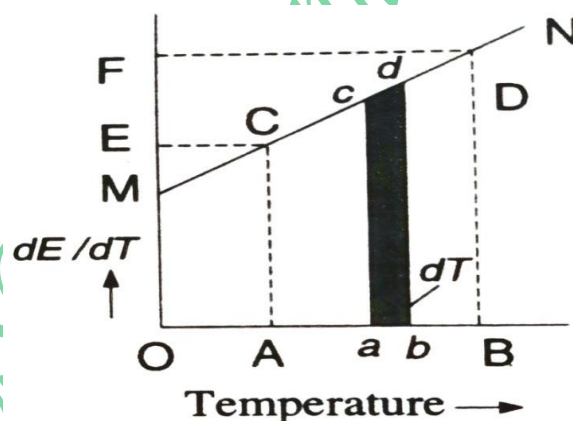
Similarly, peltier coefficient at the cold junction ( $T_1$ ) is

$$\pi_1 = T_1 \left(\frac{dE}{dT}\right)_{T_1} = OA \times AC = \text{Area } OACE$$

$\pi_1$  and  $\pi_2$  give the peltier emf's at  $T_1$  and  $T_2$  respectively.

Peltier emf between temperatures  $T_1$  and  $T_2$  is

$$E_p = \pi_2 - \pi_1 \quad \text{i.e. } \text{area } OBDF - \text{area } OACE = \text{Area } ABDFECA$$



Figure(2)

