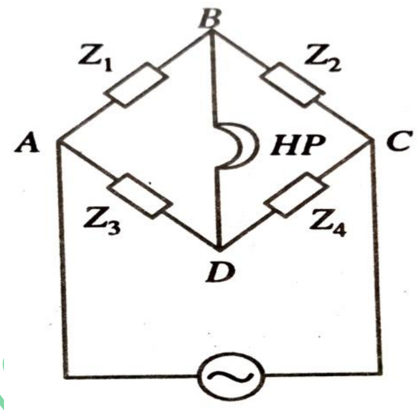
Unit II: D. C. and A.C. Bridges

Syllabus: *D.C. Wheatstone Bridge and its demerits (qualitative discussion without derivation). Theory of low resistance measurement using Kelvin's double bridge method. Measurement of inductance, Theory of Maxwell's bridge and Anderson's bridge. Comparison of capacities of two condensers by de Sauty's method.*

Whetstone's bridge:

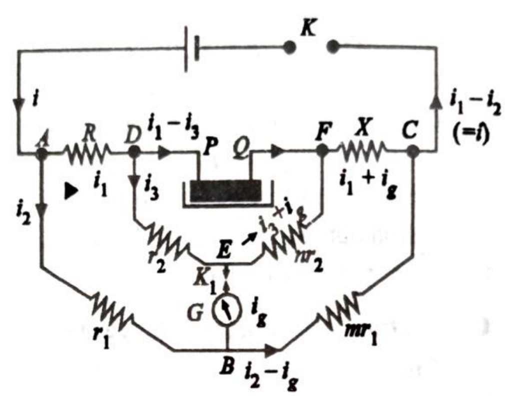


The Wheatstone bridge principle is also applicable to A.C. networks. The only modification is that here complex impedances and currents are used instead of resistances. The null point determined with the help of a pair of head phones. At balance, the points B and D are at the same potential and the head phone HP gives a minimum sound. The condition of balance for the bridge shown in fig is given by $\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$

Where Z_1, Z_2, Z_3 and Z_4 are the vector impedances of the branches. The phase balance condition is adjusted by changing reactance's (inductances and capacitances) in the arm.

Theory of low resistance measurement using Kelvin's double bridge method:

The circuit diagram of the bridge is shown in fig. ABC and DEF are two coils. B and E are contact points which divide the resistance of the respective coils in the ratio $r_1 : mr_1$ and $r_2 : nr_2$. A low resistance galvanometer is connected between B and E. A standard low resistance R is connected between A and D. The unknown low resistance X is connected between F and C. The resistances R and X are connected together through a mercury cup PQ. The other ends of R and X are connected to a battery. The distribution of currents in the branches is as shown in figure.



Applying Kirchhoff's second law to meshes ABED and BCFE, we get

$$i_2 r_1 + i_g G - i_3 . r_2 - i_1 R = 0 \ldots\ldots\ldots\ldots (1)$$

And $$(i_2 - i_g) m r_1 - (i_1 + i_g) X - (i_3 + i_g) n r_2 - i_g G = 0 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The value of R is suitably adjusted so that the galvanometer gives no deflection when the key K_1 is pressed, i.e., $i_g = 0$. Then Eqs. (1) and (2) become,

$$i_1 R = i_2 r_1 - i_3 r_2 \ldots\ldots\ldots\ldots\ldots (3)$$

And $$i_1X = i_2 .m r_1 - i_3nr_2 = 0 \dots\dots\dots\dots\dots .(4)$$

Let us put $\frac{X}{R} = m = n$, Then Eqqs. (4) reduces to

$$i_1mR = i_2mr_1 - i_3 \ mr_2 \quad or \quad i_1R = i_2r_1 - i_3r_2 \dots\dots\dots(5)$$

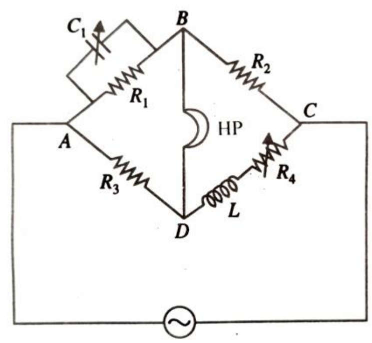
Eq. (5) is the same as Eq. (3). Thus Eqs. (3) and (4) become identical if we put $\frac{X}{R} = m = n$ So the two coils must be divided in the same ratio as regards their resistances in order to get zero deflection in the galvanometer. Hence X = mR. If the ratio m is given, the value of X can be calculated.

Maxwell's Bridge [For determination of self-inductance of a coil]

The circuit of Maxwell's L-C bridge for the determination self-inductance is shown in fig. The coil whose inductance L is to be determined is placed in the arm CD, in series with a variable non-inductive resistance R_4. Arms AB, BC and AD contain non-inductive resistance R_1, R_2 and R_3 respectively. A standard variable capacitor C_1 is connected in parallel with resistance R_1 in the arm AB.

Let Z_1, Z_2, Z_3 and Z_4 be the impedances of branches AB, BC, AD and DC respectively.

Here, $$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{1/j\omega C_1} = \frac{1}{R_1} + j\omega C_1$$



$$\text{or} \quad Z_1 = \frac{R_1}{1 + j\omega C_1 R_1}$$

$Z_2 = R_2$, $Z_3 = R_3$ and $Z_4 = R_4 + j\omega L$

According to the condition of balance,

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

$$\frac{R_1}{(1 + j\omega C_1 R_1)R_2} = \frac{R_3}{(R_4 + j\omega L)}$$

Or $$R_4R_1 + j\omega LR_1 = R_3 R_2 + j\omega C_1 R_1 R_2 R_3$$

Equating real and imaginary parts, we get

$$R_4R_1 = R_2R_3 \Rightarrow \quad \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\omega LR_1 = \omega C_1 R_1 R_2 R_3$$

And $$L = C_1 R_2 R_3$$

Eqn. (1) and (2) are the two balance conditions which are independent of one another.

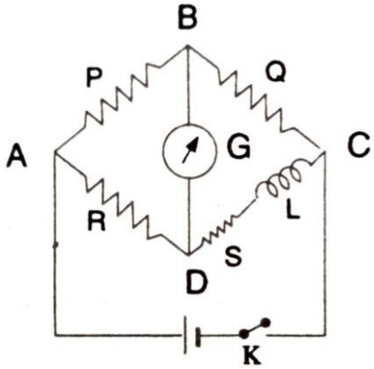
When using the bridge, R_1/R_2 is kept a simple ratio. R_4 is varied until the head phone gives minimum sound. Now the potentials at B and D are equal in magnitude. Then the capacitance C_1 is varied until the sound in the headphone reduces to a further minimum. The experiment may be repeated varying the values of the resistances.

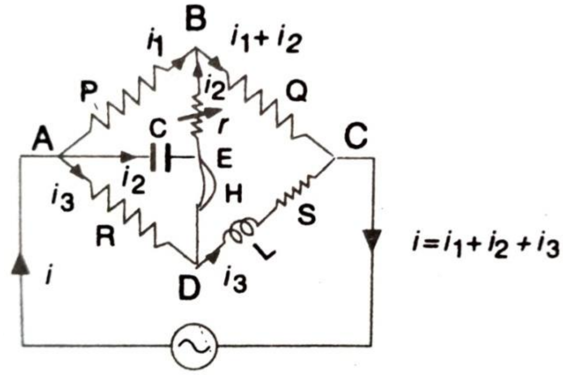
Determination of Self-inductance by Anderson's Bridge Method:

The experiment is performed in two stages.

(a)D.C balance: The circuit connections are shown in fig. The given coil of self inductance L and resistance S is connected in arm DC. The ratio arms P and Q are fixed to ratio 1 : 1 . The resistance R is adjusted for balance. This gives the approximate value of resistance S of the coil. The experiment is repeated by making the P : Q ratio to be 10 : 1 and 100 : 1. The accurate value of D.C resistance of the coil is found by the Wheatstone's bridge relation,

$$\frac{\mathbf{P}}{\mathbf{Q}} = \frac{\mathbf{R}}{\mathbf{S}} \quad \Rightarrow \quad \mathbf{S} = \mathbf{R}\frac{\mathbf{Q}}{\mathbf{P}}$$





(b) A.C. balance: An ac source (oscillator) is connected between A and C. A variable non-inductive resistance r is connected in series with a capacitance C and this combination is connected in parallel with the arm AB. A headphone H is connected between E and D. The resistance r is adjusted until minimum sound is heard in the headphone. The value of L is calculated using the formula

$$\mathbf{L = C\,[RQ + r(R + S)]}$$

Theory. Let the instantaneous currents in the different arms be as shown in fig.

At the time to balance(i.e., no current through headphone),

potential at E = potential at D.

Applying Kirchoff,s *II* law, we have

(i) for mesh ABEA,

$$\mathbf{i_1 P - i_2\left[r + \frac{1}{j\omega C}\right] = 0}$$

Or

$$i_1 = \frac{1}{P}\left[r + \frac{1}{j\omega C}\right] i_2 \;\ldots\ldots\ldots\ldots(1)$$

(ii) for mesh AEDA,

$$\frac{\mathbf{i_2}}{\mathbf{j\omega C}} - \mathbf{i_3 R = 0} \quad \Rightarrow \quad i_3 = \frac{1}{j\omega CR}\, i_2 \;\ldots\ldots\ldots\ldots(2)$$

(iii) for mesh BCDB,

$$(i_1 + i_2)Q - i_3(S + j\omega L) + i_2\,\mathrm{r} = 0$$

$$Or \qquad i_1 Q + i_2\,(Q + \mathrm{r}) - i_3(S + j\omega L) = 0 \;\ldots\ldots\ldots\ldots(3)$$

Here $\frac{1}{j\omega C}$ **and** $j\omega L$ are the impedances offered by capacitor C and inductance L respectively. ω is the angular frequency of applied a.c.

Sustituting Eqs. (1) and (2) in Eqs. (3) , we get

$$\frac{Q}{P}\left[r+\frac{1}{j\omega C}\right]i_2+i_2\,(Q+\ r)-\frac{S+\ j\omega L}{j\omega CR}i_2=0$$

Or $$\frac{Q}{P}\left[r+\frac{1}{j\omega C}\right]+(Q+\ r)-\frac{S}{j\omega CR}+\frac{L}{CR}=0\ \ldots\ldots\ldots\ldots\ldots..(4)$$

Equating the real and imaginary parts separately to zero, we get

$$\frac{Q}{P}r+Q+r-\frac{L}{CR}=0\ \ldots\ldots\ldots\ldots\ldots.(5)$$

And $$\frac{Q}{P\omega C}-\frac{S}{\omega CR}=0\ \ldots\ldots\ldots\ldots\ldots(6)$$

Eq. (6) gives $\frac{P}{Q}=\frac{R}{S}$ **Or** $S=R\frac{Q}{P}$ $\ldots\ldots\ldots\ldots\ldots\ldots(7)$

This is condition for D.C. balance.

From Eq. (5), we get

$\frac{L}{CR}=\frac{Q}{P}r+Q+r$ Or $L=CR\,[\frac{Q}{P}r+Q+r]$

Or $L=C[\,R\frac{Q}{P}r+RQ+Rr]$ Or $L=C[\,Sr+RQ+Rr]$

Or $$L=C[\,RQ+r\,(R+S)]$$

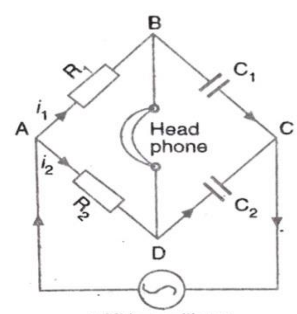
Comparison of Capacitance by de- Sauty's Bridge:

This is a simple method for comparing two capacitances. Connections are made as in fig. C_1 and C_2 are the two capacitances to be compared. R_1 and R_2 are variable, non – inductive resistances. Audio frequency oscillator is connected between A and C, and headphone between B and D.

When the bridge is balanced,

$\frac{Z_1}{Z_2}=\frac{Z_3}{Z_4}$ **Or** $\frac{Z_1}{Z_3}=\frac{Z_2}{Z_4}$ **Here** , $Z_1=R_1$, $Z_2=\frac{1}{j\omega C_1}$, $Z_3=R_3$, $Z_4=\frac{1}{j\omega C_2}$

$\therefore\ \frac{R_1}{R_2}=\frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_2}}$ **Or** $\frac{C_2}{C_1}=\frac{R_1}{R_2}$



$$C_2=\frac{R_1C_1}{R_2}\ \ldots\ldots\ldots\ldots.(1)$$

If C_1 is known, C_2 can be calculated.

Eqn. (1) is the only equation to be satisfied at balance.

Hence R_1 and R_2 are suitably variable until the sound in the headphone is inaudible.