# UNIT I:  FRAME OF REFERENCE

Meaning of event, observer and frames of reference

- **Event:** An event is something that happens at a particular point in space and at a particular instant of time, independent of the reference frame. Thus it has both a position and time of occurrence.

  **Eg:** A collision between two particles, a sudden flash of light, etc.
- **Observer:** An observer is a person or equipment to observe and take measurements about the event. The observer is supposed to have with him scale, clock and other devices needed to observe the event.
- **Frame of reference:** It is a system of coordinate axes which defines the position of a particle or an event in two or three dimensional space. To make a quantitative measurement of an event occurring in space, the observer must set up a coordinate system or a frame of reference. For convenience, a cartesian coordinate system with the observer at the origin is generally used to specify the event in space. In general, if $\vec{r}$ is the position vector of a particle in space with respect to the origin, then

$$\vec{r} = x\hat{\imath} + y\,\hat{\jmath} + z\hat{k}$$

where x, y and z are the co-ordinates along three perpendicular axes and $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ are the unit vectors along the three axes.

For the exact identification of an event in space, we must specify not only its position in space, but also its time of occurrence. A frame of reference with four coordinates x, y, z and t is referred to as **space-time reference frame**.

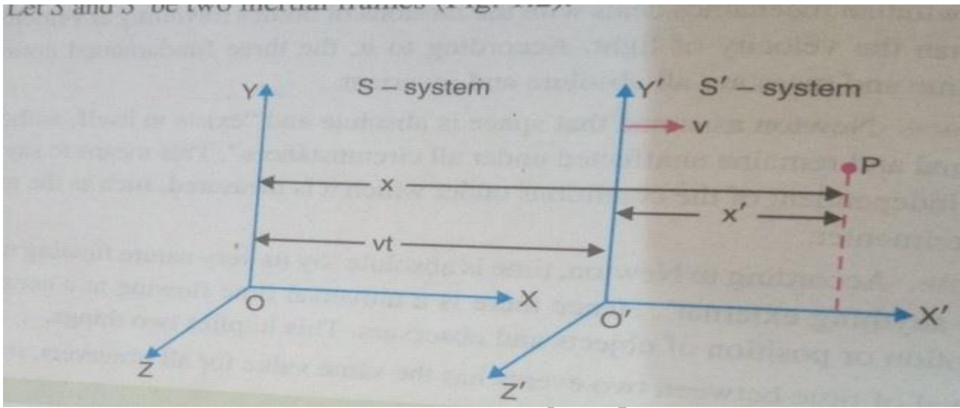
**Inertial Frame of Reference:**

Newton's first two laws of motion do not hold good in every reference frame. For example, a body at rest in one reference frame may not appear to be so in another reference frame; it may appear to be moving in a circle if observed in a reference frame rotating with respect to the first. The first two laws hold good in certain special reference frames called **inertial frames of reference.** Thus an inertial frame of reference or an inertial system is defined as one in which Newton's first two laws of motion hold good. Such a reference frame is also called **Newtonian or Galilean reference frame.** It is also termed as non-accelerating and hence non-rotating. Any other reference frame moving along a linear path with a uniform velocity relative to an inertial frame is also an **inertial frame**. An observer in an inertial frame is called an **inertial observer**.

**Galilean transformations:** Let us now discuss briefly the relativity of physical quantities in classical or Galilean Newtonian physics and know how the events in one reference frame should appear to an observer situated in another reference frame moving with some constant velocity relative to the first.

The transformation of coordinates of a particle from one frame of reference to another is called **Galilean transformation. Set of equations are known as Galilean transformation equations.**

**Transformation of position:** Let us consider one of the reference frame S with origin O to be at rest and the other reference frame $S^1$ with origin $O^1$ moving relative to O with a uniform velocity v along the positive X-direction. We assume that v<<c. Let the origin of the two frames of reference coincides at t=0.




Suppose some event is occurring in space at point P. Let an observer O at rest in the reference frame S determine the position of the event by the coordinates (x, y, z) and let this occur at time t as measured by a clock in S. Let the observer at $O^1$ in the moving frame $S'$ assign to the same event by the coordinates x', y', z' and t' the time being measured by a clock moving along with S'. Let us now see how the measurements of the coordinates of the same event in the two systems are related to each other. In time t, the frame $S'$ covers a distance $OO'$ = vt along the positive X-direction. Now x represents the distance OA whereas $x^1$ represents the distance O'A. Hence,

$$\mathbf{x' = x - vt} \quad \text{---------(1)}$$

As there is no relative motion along Y and Z directions, we have

$$\mathbf{y' = y} \text{-------------} \quad (2)$$

and

$$\mathbf{z' = z} \text{ -------------} \quad (3)$$

Further, according to classical relativity, time is supposed to be absolute' or 'universal' and can be defined independent of any particular frame of reference. Thus believing in the concept of absolute time, it follows that, $\mathbf{t' = t}$ ----- (4)

If the frame $S'$ is considered to be stationary, the observer $O'$ will find the frame S moving with a velocity v along negative X-direction so that the measurements in the frame $S'$ can be expressed in terms of those in the frame S by merely changing the sign of v. Summarizing the above transformations, we have

| | | |
|---|---|---|
| $x' = x - vt$ | | $x = x' + vt$ |
| $y' = y$ | | $y = y'$ |
| $z' = z$ | and | $z = z'$ |
| $t' = t$ | | $t = t'$ |

--------(5)

Eq. (5) are known as **Galilean transformations**

**Transformation of distance:** From Galilean transformation equations, it follows that the time interval between the occurrences of two events and the space interval between two points is the same for each inertial observer. Let us suppose that two events A and B occur at time $t_A$ and $t_B$ as measured by a clock in the frame S and at times $t^1_A$ and $t^1_B$ measured by a clock in $S^1$. Since time is supposed to be absolute in classical physics, we have $t_A = t^1_A$ and $t_B = t^1_B$.

Or $$(t_B - t_A) = (t^1_B - t^1_A) \qquad \text{------ (6)}$$

Further, suppose that the distance between two objects such as two birds flying directly over X – X' axis and at the same height are $x_A$ and $x_B$ as observed by the observer O in the frame S at time t and are $x^1_A$ and $x^1_B$ as simultaneously observed by $O^1$ in the frame $S^1$. According to Eq. (5)

$$x^1_A = x_A - vt \qquad \text{and} \qquad x^1_B = x_B - vt$$

Subtracting, we have $$(x^1_B - x^1_A) = (x_B - x_A) \qquad \text{----- (7)}$$

Eqs. (6) and (7) show that the time interval between the occurrences of two events and the space interval between two points are invariant and the same for all inertial frames.

**Transformation of velocity:** Let us now express the velocity and the acceleration of a body as measured by an observer in the frame $S^1$ in terms of those measured in the frame S. The position of a moving object in space is a function of time so that the velocity can be determined by differentiating the position with respect to time. From Eq. (5)

$$\therefore \qquad \frac{dx'}{dt} = \frac{dx}{dt} - v$$

Since t = t', the operations $\frac{d}{dt}$ and $\frac{d}{dt'}$ are identical so that

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v \quad \textit{Similarly} \quad \frac{dy'}{dt'} = \frac{dy}{dt} \textit{and} \quad \frac{dz'}{dt'} = \frac{dz}{dt}$$

But $\frac{dx}{dt} = u_x$ i.e., the x-component of the velocity measured in the frame S, and $\frac{dx'}{dt'} = u^1_x$ i.e., the x-component of the velocity in the frame $S^1$ and so on. Thus,

$u^1_x = u_x - v$ , $u^1_y = u_y$ and $u^1_z = u_z$ ……………(8)

If the relative velocity v is such that it has components in all the three directions, Eq. (8) many be written in more general vector form as

$$\vec{u'} = \vec{u} - \vec{v} \qquad \text{………..(9)}$$

Eq. (9) is known as classical velocity transformation equation. It follows from this equation that different observers in relative motion assign different values to the velocity of a particle and that the velocities differ by the relative velocity of the two observers. **Thus the velocity is not invariant under Galilean transformation.**

**Transformation of acceleration:** To obtain acceleration transformation, let us differentiate velocity transformation equations with respect to time. Thus

$$\frac{d}{dt'}(u'_x) = \frac{d}{dt}(u_x - v) = \frac{d}{dt}(u_x) \qquad (\because v\ is\ constant)$$

$$\frac{d}{dt'}(u'_y) = \frac{d}{dt}(u_y) \quad \text{and} \frac{d}{dt'}(u'_z) = \frac{d}{dt}(u_z)$$

Hence, $a'_x = a_x$ , $= a'_y = a_y$ and $a'_z = a_z$ or in general $\boxed{\overrightarrow{a'} = \vec{a}}$

It thus follows that the acceleration of a particle is the same in all inertial frames moving relative to each other with constant velocity. **Thus acceleration is invariant under Galilean transformation.**

**Newtonian Relativity:** For an observer in a frame of reference moving with uniform velocity such as a ship, all the phenomena observed or all the experiments performed are exactly the same as if the frame was not in motion. One can play badminton or tennis aboard a ship without ever knowing that the ship is in motion regardless of its velocity so long as it is uniform. This is the concept of Newtonian relativity. Thus according to Newtonian relativity, the laws of mechanics have exactly the same form in all inertial frames moving with uniform velocity relative to each other and no experiment carried out entirely in one inertial frame can tell what the motion of that frame is with respect to any other inertial frame. Let us prove it below by considering some mechanical laws.

**Newton's laws of motion:** Let us consider Newton's second law of motion which included in it the first law and is thus the real law of motion. Let a force $\vec{F}$ act upon a body of mass m in an inertial frame S. If $\vec{a}$ is the acceleration produced, we have, according to second law, $\vec{F} = \overline{m\vec{a}}$. Let F' be the corresponding force acting in another reference frame S' moving with a velocity $\vec{v}$ relative to the first. Assuming mass to be independent of velocity, we have $\vec{F} = \overline{m\vec{a}}$ where $\vec{a}$ is the acceleration produced in the frame S'. But according to Galilean transformation, $\vec{a} = a^1$ . Therefore, $\vec{F} = \vec{F}$, indicating that the second law is invariant in Galilean transformation.

**Law of conservation of momentum:** According to the law of conservation of momentum, when a number of particles collide against each other, the total momentum before the collision is the same as the total momentum after the collision, irrespective of whether the collision is elastic or inelastic.

Let $m_1$ and $m_2$ be the masses of the two particles moving with velocities $\overrightarrow{u_1}$ and $\overrightarrow{u_2}$ in the inertial frame S let $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$be the velocities of the particles after the collision. Then according to the law of conservation of momentum,

$$\boldsymbol{m_1\overrightarrow{u_1} + m_2\overrightarrow{u_2} = m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2}} \qquad \dots\dots\dots\dots(\mathbf{12})$$

Let $S^1$ be another reference frame moving with a uniform velocity $\vec{v}$ with respect to the frame S. Let $u_1^1$ and $u_2^1$ be the velocities before the collision and $v_1^1$ and $v_2^1$ be the velocities after the collision observed in the frame $S^1$ Then

$$u_1^1 = u_1 - v \quad \text{or} \quad u_1 = u_1^1 + v$$

Similarly, $u_2 = u_2^1 + v$ ; $v_1 = v_1^1 + v$ and $v_2 = v_2^1 + v$

Substituting the values of $u_1$, $u_2$, $v_1$ and $v_2$ in Eq. (12), we have

$$m_1(u_1^1 + v) + m_2(u_2^1 + v) = m_1(v_1^1 + v) + m_2(v_2^1 + v)$$

$$or \quad m_1\overrightarrow{u'_1} + m_1\vec{v} + m_2\overrightarrow{u'_2} + m_2\vec{v} = m_1\overrightarrow{v'_2} + m_2\vec{v} + m_2\overrightarrow{v'_2} + m_2\vec{v}$$

$$or \quad m_1\overrightarrow{u'_1} + m_2\overrightarrow{u'_2} + (m_1 + m_2)\vec{v} = m_1\overrightarrow{v_1'} + m_2\overrightarrow{v'_2} + (m_1 + m_2)\vec{v}$$

$$\boldsymbol{or \quad m_1\overrightarrow{u'_1} + m_2\overrightarrow{u'_2} = m_1\overrightarrow{v_1'} + m_2\overrightarrow{v'_2}} \quad \ldots\ldots..(\mathbf{13})$$

Eq. (13) shows that the law of conservation of momentum holds goods in all inertial frames, i.e. it is invariant in Galilean transformation.

**Law of conservation of Energy:** According to the law of conservation of energy , in an elastic collision ,when a number of particles collide against each other, the total energy before the collision is the same as the total energy after the collision.

Let $m_1$ and $m_2$ be the masses of the two particles moving with velocities $\overrightarrow{u_1}$ and $\overrightarrow{u_2}$ in the inertial frame S let $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ be the velocities of the particles after the collision. Then according to the law of conservation of momentum,

$$\boldsymbol{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2} \quad \ldots\ldots\ldots\ldots(\mathbf{14})$$

Let $S^1$ be another reference frame moving with a uniform velocity $\vec{v}$ with respect to the frame S. Let $u_1^1$ and $u_2^1$ be the velocities before the collision and $v_1^1$ and $v_2^1$ be the velocities after the collision observed in the frame $S^1$ Then, $u_1^1 = u_1 - v$ or $u_1 = u_1^1 + v$

Similarly, $u_2 = u_2^1 + v$ ; $v_1 = v_1^1 + v$ and $v_1 = v_2^1 + v$

Substituting the values of $u_1$, $u_2$, $v_1$ and $v_2$ in Eq. (14), we have

$$\frac{1}{2}m_1(u_1^1 + v)2 + \frac{1}{2}m_2(u_2^1 + v)2 = \frac{1}{2}m_1(v_1^1 + v)2 + \frac{1}{2}m_2(v_2^1 + v)2$$

$$\frac{1}{2}m_1\,(u_1^{12} + 2u_1^1v + v^2) + \frac{1}{2}m_2\,(u_2^{12} + 2u_2^1v + v^2)$$

$$= \frac{1}{2}m_1\,(v_1^{12} + 2v_1^1v + v^2) + \frac{1}{2}m_2\,(v_2^{12} + 2v_2^1v + v^2)$$

$$\frac{1}{2}m_1u_1^{12} + \frac{1}{2}m_2u_2^{12} + (m_1u_1^1 + m_2u_2^1)\text{v} = \frac{1}{2}m_1v_1^{12} + \frac{1}{2}m_2v_2^{12} + (m_1v_1^1 + m_2v_2^1)\text{v}$$

$$\boldsymbol{\frac{1}{2}m_1u_1^{12} + \frac{1}{2}m_2u_2^{12} = \frac{1}{2}m_1v_1^{12} + \frac{1}{2}m_2v_2^{12}} \ldots\ldots\ldots\ldots.(\mathbf{15})$$

Eq. (15) shows that the law of conservation of energy holds goods in all inertial frames, i.e. it is invariant in Galilean transformation.